Programmed Problem 1 for linear dielectrics

This is problem 1 for test 2. It is Due Wednesday April 9 by 4 pm.

You may use your notes, and our web site. You may not discuss with others except me. This is a pledged activity.

A solid conducting sphere of radius a has a total charge Q.

(1) What is the surface charge density?

(2) What is the electric field inside the sphere?

(3) What is the electric field outside the sphere?

(4) What is D outside the sphere? Note: use Gauss's law here.

(5) How are D, E and P related?

(5a) What is P?

(6) Now, place a linear dielectric around the sphere up to a radius b.

What is D?

(7) How is P related to E?

(8) What is the electric field inside the dielectric in terms of Q, r, etc? Note: use (5) and (6), and (7) here.

(9) Why can you not yet find E?

Gauss gives: $\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

But Q owes to both free and bound charges. We need to find these.

(10) How do you obtain the bound volume charge density from the polarization?

(10a) What is the bound volume charge density here? Hint: zero.

(11) How do obtain surface charge density from polarization?

(12) at the surface r=a, convince yourself that n for the dielectric points in the -r direction.

(13) Find the bound surface charge density at r=a.

(13a) Find the total bound surface charge at r=a.

(14) at the surface r=b, convince yourself that n for the dielectric points in the +r direction.

(15) Find the bound surface charge density at r=0.

(15a) Find the total bound surface charge at r=b.

(16) Add your results from (13a) and from (15a) to find the total bound charge.

(17) Use Gauss's law now to find the electric field outside the sphere.

A solid conducting sphere of radius a has a total charge Q.

(1) What is the surface charge density?

$$\sigma=\frac{Q}{4\pi a^2}$$

(2) What is the electric field inside the sphere?
E=0

(3) What is the electric field outside the sphere?

$$\vec{E}=\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

(4) What is D outside the sphere?

$$\vec{D}=\frac{Q}{4\pi r^2}\hat{r}$$

How do we know?

$$\oiint \vec{D}\cdot d\vec{A}=Q_{enc}\Rightarrow\vec{D}=\frac{Q}{4\pi r^2}\hat{r}$$

(5) what is the polarization?

$$\vec{D}=\epsilon_0\vec{E}+\vec{P}\Rightarrow\vec{P}=\vec{0}$$

(6) Place a linear dielectric around the sphere up to a radius b.
What is D?
D is exactly the same so long as the free surface charge on the sphere is unchanged.

(7) How is P related to E?

$$\vec{P}=\epsilon_0\chi_E\vec{E}$$

(8) What is the electric field inside the dielectric in terms of Q, r, etc?

$$\vec{D}=\epsilon_0\vec{E}+\epsilon_0\chi_E\vec{E}=\epsilon_0(1+\chi_E)\vec{E}\Rightarrow\vec{E}=\frac{\vec{D}}{\epsilon_0(1+\chi_E)}=\frac{Q}{4\pi\epsilon_0(1+\chi_E)r^2}\hat{r}$$

(9) What is the electric field outside the dielectric?

$\oiint \vec{E}\cdot d\vec{A}=\frac{Q_{enc}}{\epsilon_0}$ But Q owes to both free and bound charges. We need to find these.

Bound charges: $\rho_b=-\vec{\nabla}\cdot\vec{P}=0$

Surface charges: $\sigma_b=[\vec{P}\cdot\hat{n}]_{surface}$

at r=a, $\hat{n}=-\hat{r}$ At the surface r=a, what is P?

$$\vec{P}=\frac{\epsilon_0\chi_E}{4\pi\epsilon_0(1+\chi_E)a^2}\hat{r}\Rightarrow\sigma_{b,a}=-\frac{\chi_E Q}{4\pi(1+\chi_E)a^2}$$

What is the total bound charge on the surface?

$$Q_{b,a} = \frac{-\chi_E Q}{1+\chi_E}$$

at r=b, $\hat{n} = \hat{r}$ At the surface r=b, what is P?

$$\vec{P} = \frac{\epsilon_0 \chi_E}{4\pi\epsilon_0 (1+\chi_E) b^2} \Rightarrow \sigma_{b,b} = \frac{\chi_E Q}{4\pi(1+\chi_E) b^2}$$

What is the total bound charge at the surface b?

$$Q_{b,b} = \frac{\chi_E Q}{(1+\chi_E)}$$

What is the total bound charge on the dielectric?

$$Q_b = Q_{b,a} + Q_{b,b} = 0$$

What is the electric field outside the dielectric?

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$