Example 2.6

A hollow sphere of charge Q of radius a is centered at the origin. Find the potential everywhere.

$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

Outside the sphere, Q acts as if it were concentrated at the center of the sphere.
I've used here the fact that

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin(\theta)\,d\phi\,\hat{\phi}$$

$$V = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r} = \frac{Q}{4\pi\epsilon_0 r} = \frac{kQ}{r}$$

Note that $d\vec{l} = -d\vec{r}$ here because you come in from infinity.

At the surface of the sphere, $V = \frac{kQ}{a}$

Inside the sphere, since the electric field is zero, and breaking up the integral into 2 parts shows that the potential inside is exactly the potential at the surface even though E is zero inside the sphere.

Guess I've killed this one off.