

**Instructions: You have a total of 50 minutes to complete this test.**

**Answer each of the following questions completely, showing full details with correct SI units.**

Time Start \_\_\_\_\_ Time finish \_\_\_\_\_ Pledged \_\_\_\_\_

**Do not discuss any aspect of this test with anyone until I return the test.**

**Constants:**  $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$

**[1]** A circuit consists of an inductor ( $L=10 \times 10^{-4} \text{ h}$ ) and a capacitor ( $C=10 \times 10^{-4} \text{ f}$ ) in series.

**(a)** Calculate the resonance frequency ( $f_0$ ) of this circuit.

$f_0 =$  \_\_\_\_\_

**(b)** Another circuit consists of an inductor ( $L=1 \times 10^{-5} \text{ h}$ ), a capacitor ( $C=1 \times 10^{-5} \text{ f}$ ) and a resistor ( $R=10 \text{ } \Omega$ ) in series. If the circuit is operated at  $f=100 \text{ Hz}$ , calculate the following 3 quantities:

$X_L =$  \_\_\_\_\_

$X_C =$  \_\_\_\_\_

$Z =$  \_\_\_\_\_

**(c)** A transformer consists of a primary coil with 100 turns and a secondary coil with 50 turns. If an input voltage of 10 V (RMS) AC is applied to the primary side, what is the secondary voltage?

$V =$  \_\_\_\_\_

**(d)** A transformer consists of a primary coil with 100 turns and a secondary coil with 50 turns. If an input voltage of 10 V (RMS) DC is applied to the primary side, what is the secondary voltage?

$V =$  \_\_\_\_\_



**[2]** An ideal solenoid has a total length  $h$  and the interior cross sectional area is  $A$  with a total of  $N$  coils and a turn density  $n=N/h$ . The current  $I$  is injected into the solenoid at the top and exits at the bottom as shown. Note that in the image to the left, dashed portions are behind while solid portions are in front.

**(a)** Circle the correct answer for the direction of the magnetic field inside the solenoid.

$+\hat{z}$

$-\hat{z}$

**(b)** Find the **magnitude** of the **magnetic** field inside the solenoid near the center. **Your answer involves  $n$ ,  $I$  and a constant.**

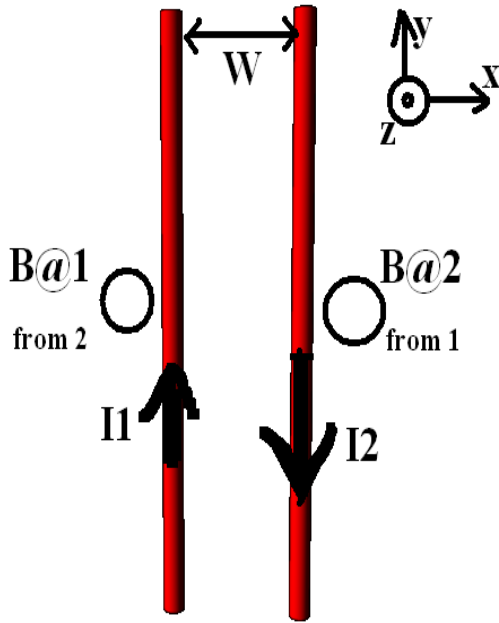
**(c)** Assuming the magnetic field is uniform throughout the solenoid, calculate the magnitude of the total magnetic flux in the solenoid when the current  $I$  is flowing. Your answer involves  $n$ ,  $I$  and the interior volume of the solenoid.

**(d)** Calculate the inductance of the solenoid in terms of  $n$ ,  $A$  and  $h$ .

**(e)** Calculate the **total magnetostatic energy** of the solenoid. Here, express your answer in terms of a constant,  $B$ ,  $A$  and  $h$ . You may assume the solenoid is ideal.

**(f)** Provide numerical answers for (b), (c), (d) and (e) **together with correct SI units** for the case  $I=20\text{A}$ ,  $n=1000/\text{m}$ ,  $A=0.2\text{m}^2$ ,  $h=1\text{ m}$ .

**(f:b)** \_\_\_\_\_ **(f:c)** \_\_\_\_\_ **(f:d)** \_\_\_\_\_ **(f:e)** \_\_\_\_\_



**[3]** Two long wires (each with the same length  $L_1, L_2$ ) carry currents  $I_1$  and  $I_2$  in the directions shown. In the coordinate system indicated,  $z$  is out of the paper. You may assume the wires are long enough so that they may be treated as ideal. **Note that a dot represents a vector coming out of the page while a cross is a vector pointing into the page.**

**(a)** In the circles provided, show the direction that the magnetic fields point for  $B$  at 1 from 2 and  $B$  at 2 from 1. Use these symbols:  $\odot$  if the field is in the  $+z$  direction and  $\otimes$  for the field directed into the  $-z$  direction.

**(b)** Calculate the magnitude of the magnetic field  $B@2$  from 1.

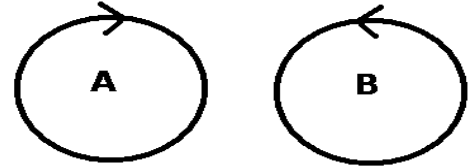
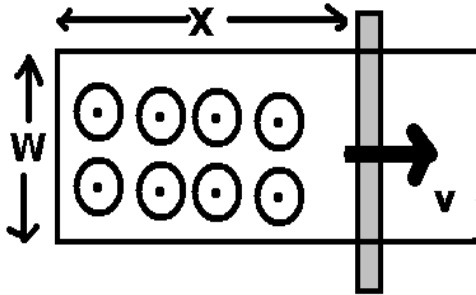
**(c)** Calculate the **vector force** on wire 2 due to wire 1. **You must include correct unit vectors here for full credit.**

**(d)** Suppose that  $I_1 = I_2 = 3A$ ,  $w = 1.0$  m and  $L_1 = L_2 = 3.0$  m. Provide numerical answers for (b) and (c) **together with correct SI units and unit vectors.**

**(d:b)** \_\_\_\_\_

**(d:c)** \_\_\_\_\_

**[4]** Consider the following situation: a conducting rail is moving with an instantaneous position given by  $x = x_0 + pt^5$  ( $x_0$  and  $p$  are constants and the rail is moving in the  $+x$  direction). In the area enclosed by the rail system, a uniform magnetic field ( $B$ ) is directed in the positive  $z$  direction:  $\vec{B} = |\vec{B}|\hat{z}$ . The rail system has a total resistance (at the end only) given by  $R$  and this value is assumed to be constant throughout this problem.



**[a]** At an instant in time, calculate the **magnitude** of the magnetic flux through the enclosed region of the system. You may assume the normal to the area of the enclosed region points into the  $+z$  direction.

**[b]** Find the **magnitude** of the induced emf in the system at any time  $t$ .

**[c]** Which direction will the induced current flow: (A or B) and why (in words). Be very clear in your answer to this question.

**[d]** Suppose  $p = 2 \text{ m/s}^5$ ,  $w = 1.0 \text{ m}$ ,  $x_0 = 100 \text{ m}$ ,  $t = 2 \text{ s}$ , and  $B = 1 \text{ T}$ . Provide numerical answers to [a] and [b] **together with correct SI units**.

**[d:a]** \_\_\_\_\_

**[d:b]** \_\_\_\_\_