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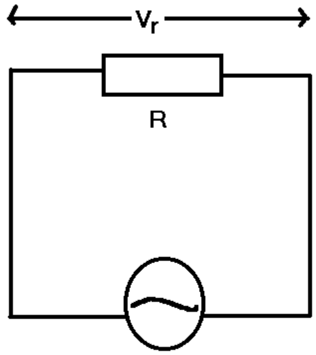
A note about notation: lowercase I p. v and i are instantaneous values.



$v=V_m \sin(\omega t)$

This analysis is valid if the circuit contains only a resistor and the applied voltage must vary as:

$$v=V_m \sin(\omega t) \quad .$$

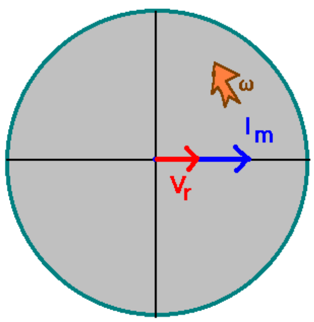
The voltage drop across the resistor, v_r is equal to the instantaneous applied voltage, v. Thus $v_r=V_m \sin(\omega t)$. The instantaneous current is equal to

$$i=\frac{v_r}{R}=\frac{V_m}{R}\sin(\omega t) \quad .$$

The **peak current** is given by $I_m=\frac{V_m}{R}$.

The instantaneous voltage drop across the resistor is

$$v_r=I_m R \sin(\omega t)$$



and the current is said to be in phase with the voltage.

We need to calculate the time-average power radiated by the resistor. The instantaneous power radiated by the resistor is given by

$$p=i^2 R$$

where i is the instantaneous current. We can calculate the time average power by using a slick trick that you have seen me use before. ...

$$<P>=<i^2 R>=I_m^2 R <\sin^2(\omega t)>=\frac{V_m^2}{R}<\sin^2(\omega t)> \quad .$$

To evaluate the time average of $<\sin^2(\omega t)>$, consider the trigonometric relation: $\sin^2\theta+\cos^2\theta=1$. The time average of 1 is 1: <1>=1, and the $\sin^2\theta$ behaves exactly like the $\cos^2\theta$ and so by symmetry, both of these must contribute equally to the time average. Thus we reach the conclusion that $<\sin^2(\omega t)>=\frac{1}{2}$. Thus,

$$<P>=\frac{V_m^2}{2R}=\frac{I_m^2 R}{2} \quad .$$

We know also that the instantaneous power is also given by $p=iv$. For AC circuits satisfying these conditions, we redefine I to be I_{rms} and V to be V_{rms} by

$$I_{rms}\equiv\frac{I_m}{\sqrt{2}} \quad \text{and} \quad V_{rms}\equiv\frac{V_m}{\sqrt{2}} \quad \text{so we can write} \quad <P>=I_{rms}V_{rms}$$

Note this is true only for the initial assumptions in this problem.!!!!

In particular if we have capacitance and inductance in the circuit, this must be modified by a factor known as the "power factor" which comes later.

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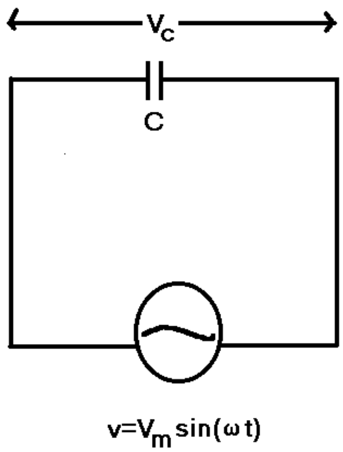


This analysis is valid if the circuit contains only a capacitor and the applied voltage must vary as:

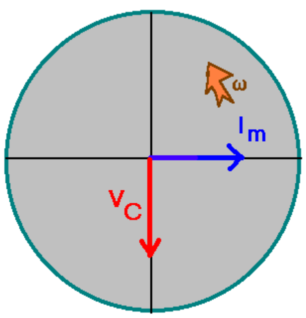
$$v = V_m \sin(\omega t) \; .$$

The instantaneous voltage drop across the capacitor is $v_C = V_m \sin(\omega t)$. The instantaneous current is then (through the magic of calculus):

$$i_C = \omega C V_m \cos(\omega t) \; .$$

Notice that the current is **not in phase** with the applied voltage. We can show this most clearly by rewriting the current as

$$i_C = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) \; .$$



The current reaches its peak value 1/4 of a cycle sooner than the voltage reaches its peak value. For this reason, in this type of circuit obeying the initial assumptions it is said that the ***current always leads the voltage across a capacitor by 90^0***.

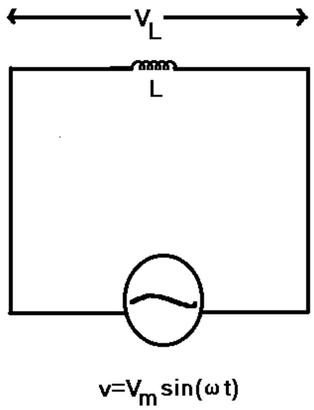
The peak current in the circuit is given by $I_m = \omega C V_m$. We want to define a new quantity called the ***capacitive reactance***:

$$X_C \equiv \frac{1}{\omega C}$$

so that the instantaneous voltage drop across the capacitor is given by

$$v_C = I_m X_C \sin(\omega t) \; .$$

Note that the units of capacitive reactance are those of Ohms! You can show this since c=Q/V and ω has units of "rad"/s so $[X_c]=[V/I]=$Ohms.



This analysis is valid if the circuit contains only an inductor and the applied voltage must vary as:

$$v = V_m \sin(\omega t) \quad .$$

The instantaneous voltage drop across the inductor is

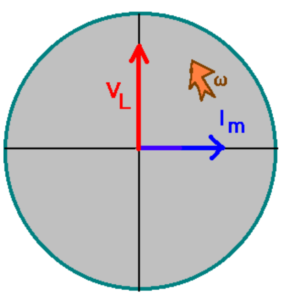
$$v - L\frac{\Delta I}{\Delta t} = 0$$

The instantaneous current is then given by (through the magic of calculus)

$$i_L = \frac{-V_m}{\omega L}\cos(\omega t)$$

We can rewrite this in terms of the sin as

$$i_L = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$



The current reaches its peak value 1/4 of a cycle later than the voltage reaches its peak value. For this reason, in this type of circuit obeying the initial assumptions it is said that the ***current always lags the voltage across an inductor by 90^0***.

The peak current through the inductor is given by

$$I_m = \frac{V_m}{\omega L}$$

We want to define a new quantity called the ***inductive reactance***:

$$X_L \equiv \omega L$$

so that the instantaneous voltage drop across the inductor is given by

$$v_L = I_m X_L \sin(\omega t) \quad .$$

Note that the units of inductive reactance are those of Ohms! You can show this since [L]=[Vt/A] and ω has units of "rad"/s so [X_c]=[V/I]=Ohms.

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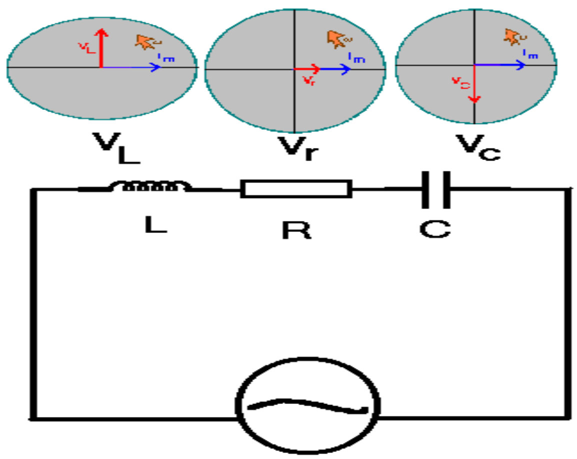


$$v = V_m \sin(\omega t)$$

This analysis is strictly valid ONLY for the series RLC circuit. For other configurations, you must consider other methods for combining the reactances.

$$\text{Reactance:} \quad X_C \equiv \frac{1}{\omega C} \qquad X_L \equiv \omega L$$

Treat the various voltages across the various elements as vectors and find the magnitude:

$$|\vec{V}| = |\vec{V}_R + \vec{V}_C + \vec{V}_L| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

In terms of the reactance, then

$$V_R = I_m R : V_L = I_m X_L : V_C = I_m X_C$$

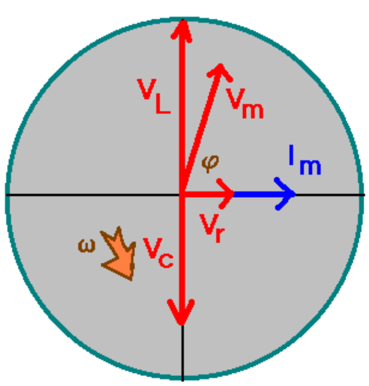
So the magnitude of V_m is given by

$$|\vec{V}| = V_m = I_m \sqrt{R^2 + (X_L - X_c)^2} = I_m Z$$

where Z is called the impendence of the circuit

and the angle that V_m makes with respect to the current is given by

$$\tan(\phi) = \frac{X_L - X_C}{R}$$



Now you can easily see that the time average power radiated by the RLC circuit is given by

$$\langle P \rangle = I_{RMS} V_{RMS} \cos(\phi)$$

where the last term is the "power factor". You will also notice that resonance occurs when $X_c = X_L$. This then gives the resonance frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$

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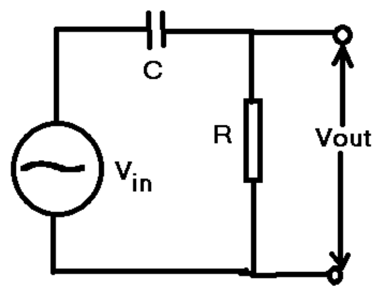
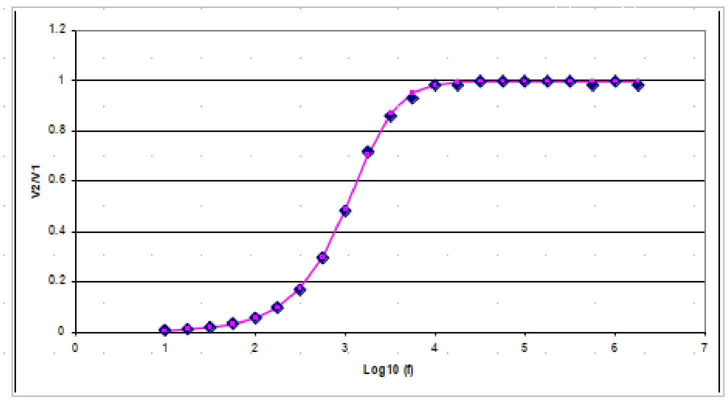
2 filter circuits: The high-pass filter and the low pass filter.
The easiest way to remember how to treat RC filter circuits is to look at the capacitive reactance, X_c. For low frequencies, capacitor acts like on open circuit. For high frequencies, the capacitor acts like a short circuit. We'll get the intermediate response here also.

1. The high-pass filter. We want to look for the ratio of V_{out} to V_{in}. This is given by the high-pass response function which is also called the circuit gain A_v:



$$A_v = \frac{V_{out}}{V_{in}} = \frac{I_m R}{I_m Z} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}$$



Notice that the HF response is that which would happen for the capacitor if it were replaced by a piece of wire.

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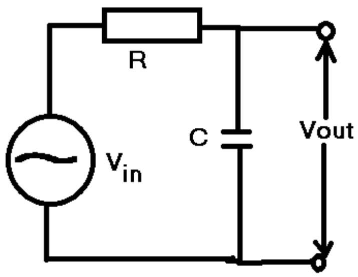
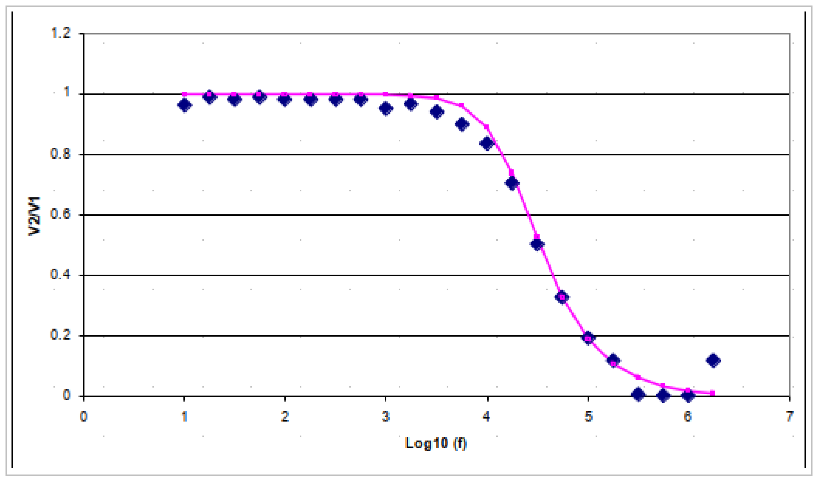
2. The low-pass filter. We want to look for the ratio of V_{out} to V_{in}. This is given by the low-pass response function which is also called the circuit gain, A_v:



$$A_v = \frac{V_{out}}{V_{in}} = \frac{I_m X_C}{I_m Z} = \frac{\left(\frac{1}{\omega C}\right)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$



Note that at high frequencies, this acts much like a short across the capacitor. Now a question for you: suppose you replace the capacitor in these two circuits with an inductor. What are the response functions in those cases?