

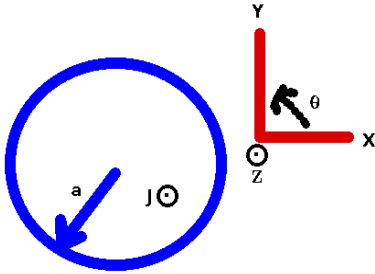
**Instructions: You have a total of 50 minutes to complete this test.**

**Answer each question completely showing complete details.**

**For complete credit you must include correct SI units with numerical answers.**

Time Start \_\_\_\_\_ Time finish \_\_\_\_\_ pledged \_\_\_\_\_

Constants:  $k = 8.987 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$ ;  $\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$ ;  $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$ ;  $\mu = 1 \times 10^{-6}$



[1] A cylindrical wire of cross sectional area  $\pi a^2$  has a radius  $a$  as shown. The wire carries current density given by  $\vec{J} = cr^3 \hat{z}$ . **Hint:** you will have to use the differential cross sectional area  $dA = 2\pi r dr$  here with appropriate limits.

[a] If the wire carries a total current  $I$ , find the constant  $c$  in terms of the radius of the wire.

[b] Find the **vector magnetic field** inside the wire ( $r < a$ ) in terms of  $r$  and  $J$ . Be sure to include the appropriate unit vector here.

[c] Find the vector magnetic field outside the wire ( $r > a$ ) in terms of  $r$ ,  $a$  and  $J$ . Again, be sure to include the appropriate unit vector here.

[d] If  $a = 2$  m, and  $c = 1$  A/m<sup>5</sup>, provide a numerical value for the magnitude of the magnetic field at the surface of the wire (where  $r = a$ ) with correct SI units.

**[2]** A parallel plate capacitor has plates of area  $A$  and separation  $d$ . On the plate located at the  $z=0$ , a surface charge density  $+\sigma$  exists while on the plate located at  $z=d$ , a surface charge density  $-\sigma$  exists. Answer the following assuming that this can be regarded as an ideal capacitor.

**(a)** Starting from Gauss's Law, show (showing complete details) how to calculate the vector electric field within the capacitor in terms of  $\sigma$ ,  $\epsilon_0$  and  $\hat{z}$ .

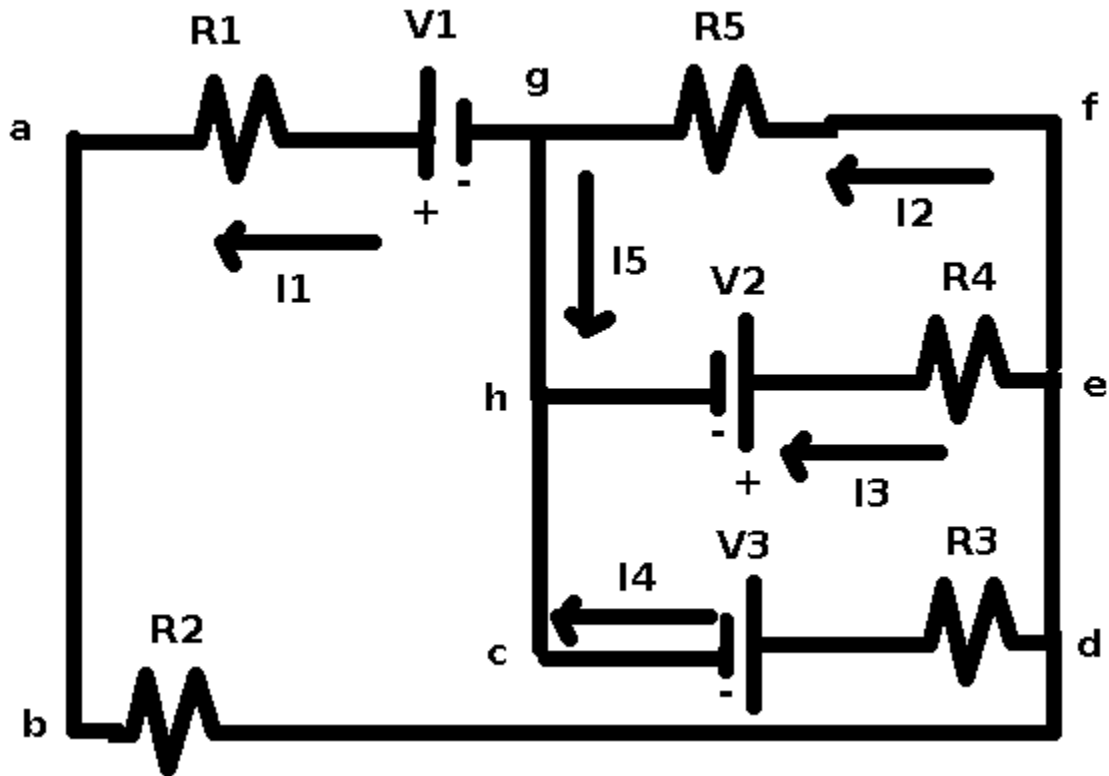
**(b)** Calculate the **magnitude** of the potential difference between the two plates in terms of  $\sigma$ ,  $\epsilon_0$  and  $d$ .

**(c)** What is the capacitance of the capacitor in terms of  $\epsilon_0$ ,  $A$  and  $d$ .

**(d)** What is the total energy stored on the capacitor in terms of  $C$  and  $V$ .

**(e)** Calculate the energy density in terms of  $\epsilon_0$  and  $E$ .

**[3]** Consider the circuit shown above. Write the Kirchoff's laws equations that result from the following:



Loop (abdchga) \_\_\_\_\_

Loop (ghefg): \_\_\_\_\_

Loop (hcdeh): \_\_\_\_\_

@g: \_\_\_\_\_

@h: \_\_\_\_\_

Suppose with the following values:

$V_1=10\text{V}$  :  $V_2=20\text{V}$  :  $V_3=30\text{V}$  :  $R_1=1\Omega$  :  $R_2=2\Omega$  :  $R_3=3\Omega$  :  $R_4=4\Omega$ , the following currents resulted:  $I_1=-2.08\text{A}$ ,  $I_2=3.44\text{A}$ ,  $I_3=-0.94\text{A}$ ,  $I_4=-4.58\text{A}$ ,  $I_5=5.52\text{A}$

Calculate the power by resistor R3 with correct SI units.

$P=$  \_\_\_\_\_

What is the interpretation of the value provided for current  $I_1$ ?

**[4]** A resistor has a resistivity of  $2 \Omega\text{m}$ . A resistor from this material is  $1 \text{ m}$  long and has a cross sectional area of  $0.1 \text{ m}^2$ . Be sure to provide correct SI units to your answers.

**(a)** Calculate the resistance of this resistor.

**(b)** If  $R_1$  has a resistance of  $10 \Omega$  and resistor  $R_2$  has a resistance of  $5 \Omega$ , calculate the resistance of the two resistors when connected in parallel.

**(c)** If  $R_1$  has a resistance of  $10 \Omega$  and resistor  $R_2$  has a resistance of  $5 \Omega$ , calculate the resistance of the two resistors when connected in series.

A capacitor has a plate area of  $1.5 \text{ m}^2$  and a plate separation of  $0.01 \text{ m}$ .

**(d)** Calculate the capacitance of this capacitor.

**(e)** If  $C_1$  has a capacitance of  $2 \mu\text{f}$  and  $C_2$  has a capacitance of  $3 \mu\text{f}$ , calculate the capacitance of the two capacitors when they are connected in parallel.

**(f)** If  $C_1$  has a capacitance of  $2 \mu\text{f}$  and  $C_2$  has a capacitance of  $3 \mu\text{f}$ , calculate the capacitance of the two capacitors when they are connected in series.

**(g)** Calculate the RC time constant when a resistor ( $R=1 \times 10^4 \Omega$ ) is connected in series with a capacitor ( $C=5 \mu\text{f}$ ).