9.4.2 Reflection at a conducting surface

You have free charges and free currents at conducting surfaces. The boundary conditions are then:

$$\epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma_{Q,f} \qquad \vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$$

$$B_1^{\perp} - B_2^{\perp} = 0 \qquad \frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = \vec{K}_f \times \hat{n}$$

Noting that n is the normal to the area and not the polarization direction and n points from medium 2 towards medium 1.

Reflection and Transmission at normal incidence to a conducting plane

The model is that in the x-y plane exists an infinite interface . In the -z region, space is non-conducting. In the +z region, space is conducting. The incident wave propagates in the +z direction. Let the electric field be polarized along the +x direction.

The **incident fields** are given by:

$$\vec{\tilde{E}}_I = \tilde{E}_{o,I} e^{i(k_1 z - \omega t)} \hat{x}$$

$$\vec{\tilde{B}}_I = \frac{1}{v_1} \tilde{E}_{0,I} e^{i(k_1 z - \omega t)} \hat{y}$$

The **reflected fields** are given by:

$$\vec{\tilde{E}}_R = \tilde{E}_{o,R} e^{i(-k_1 z - \omega t)} \hat{x}$$

$$\vec{\tilde{B}}_R = -\frac{1}{v_1} \tilde{E}_{0,R} e^{i(-k_1 z - \omega t)} \hat{y}$$

where the (-) is required since $\vec{\tilde{B}} = \frac{1}{v} \vec{\tilde{k}} \times \vec{\tilde{E}}$

The **transmitted fields** are given by:

$$\vec{\tilde{E}}_T = \tilde{E}_{o,T} e^{i(\tilde{k}_2 z - \omega t)} \hat{x}$$

$$\vec{\tilde{B}}_T = \frac{\tilde{k}_2}{\omega} \tilde{E}_{0,T} e^{i(\tilde{k}_2 z - \omega t)} \hat{y}$$

where $\tilde{k}^2 = \mu \epsilon \omega^2 + i \mu \sigma \omega$ (9.124)

Where I have introduced the complex wave number to compensate for the possible resistive nature of the conductor which will convert EM energy into Joule heating. It is also necessary to use the dispersion relation here instead of the simple velocity relation we had before.

For normal incidence, there will no perpendicular component of E. This sort-of means that $\sigma_{f,Q} = 0$. If there was somehow initially a charge for most materials this would bleed off into the inner depths of the conductor so eventually this would be true. It is also this situation: since $\vec{J} = \sigma \vec{E}$, there is no free current at this interface because it would require an infinite E. So, the formal solutions are:

$$\tilde{E}_{0,R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right) \tilde{E}_{0,I} : \tilde{E}_{0,T} = \left(\frac{2}{1+\tilde{\beta}}\right) \tilde{E}_{0,I}$$

but note now that beta is complex. If the conductor is a **perfect conductor**, beta is infinite. This shows the reflected wave has a 180 degree phase shift and the transmitted wave is zero. Thus mirrors are coated with silver to make them highly reflective.