9.2.1 EM waves in a vacuum without doing true justice

Recall Maxwell's equations:

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\epsilon_0} \quad \vec{\nabla}\cdot\vec{B}=0 \quad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \quad \vec{\nabla}\times\vec{B}=\mu_0\vec{J}+\epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

In a vacuum, there are no charges and thus no conventional currents. These become:

$$\vec{\nabla}\cdot\vec{E}=0 \quad \vec{\nabla}\cdot\vec{B}=0 \quad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \quad \vec{\nabla}\times\vec{B}=\epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

Take the curl of the last two equations:
And, by the way, we are going to use a vector theorem for curls:

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)-\nabla^2\vec{E}=\vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)=\frac{-\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)=-\epsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

So: $\nabla^2\vec{E}-\epsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}=\vec{0}$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right)-\nabla^2\vec{B}=\epsilon_0\mu_0\vec{\nabla}\times\left(\frac{\partial\vec{E}}{\partial t}\right)=\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)=-\epsilon_0\mu_0\frac{\partial^2 B}{\partial t^2}$$ so:

$$\nabla^2\vec{B}-\epsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}=0$$

These represent 3 dimensional (coupled waves) which travel with a speed of:

$$v=\frac{1}{\sqrt{\epsilon_0\mu_0}}=c=3\times10^8\,\text{m/s}$$