Maxwell's Equations (before Maxwell)

Gauss's law for electric charges: $\oiint \vec{E}\cdot d\vec{A}=\frac{1}{\epsilon_0}\iiint \rho\, d^3r \Rightarrow \vec{\nabla}\cdot\vec{E}=\frac{\rho}{\epsilon_0}$

Gauss's law for magnetic charges: $\oiint \vec{B}\cdot d\vec{A}=0 \Rightarrow \vec{\nabla}\cdot\vec{B}=0$

Faraday's law: $\oint \vec{E}\cdot d\vec{L}=-\frac{\partial}{\partial t}\oiint \vec{B}\cdot d\vec{A} \Rightarrow \vec{\nabla}\times\vec{E}=-\frac{\partial \vec{B}}{\partial t}$

Ampere's law: $\oint \vec{B}\cdot d\vec{L}=\mu_0 \oiiint \vec{J}\cdot d\vec{A} \Rightarrow \vec{\nabla}\times\vec{B}=\mu_0\vec{J}$

You have seen the demonstration of the missing element in these equations in Phy250: for steady currents, there is no problem. Here is another demonstration of the fundamental flaw:

Looking at Faraday's law, take the divergence:

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{E}\right)=-\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{B}$$

Since the divergence of a curl is zero, and B does not diverge, there is no problem.

Let's look at Ampere's law and take the divergence:

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right)=\mu_0\vec{\nabla}\cdot\vec{J}$$

Again, the divergence of a curl is zero so the left hand side is zero but the right hand side is only zero for a steady current density. In general, the current density might diverge which means that Ampere's law is incomplete.

Maxwell fixed this problem by postulating the existence of an additional term for Ampere's law which was a "new" type of current called the displacement current:

$$\vec{J}_D\equiv\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

With this correction, Ampere's law becomes:

$$\oint \vec{B}\cdot d\vec{L}=\mu_0\oiiint\left[\vec{J}+\epsilon_0\frac{\partial\vec{E}}{\partial t}\right]\cdot d\vec{A}\Rightarrow\vec{\nabla}\times\vec{B}=\mu_0\vec{J}+\epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

Which together with the Lorentz force law: $\vec{F}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$ summarize the entire theoretical content of classical electrodynamics except for material properties.

I want to show now how the continuity equation comes from Ampere's corrected law:

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right)=0=\vec{\nabla}\cdot\vec{J}+\epsilon_0\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{E}\Rightarrow\vec{\nabla}\cdot\vec{J}=-\epsilon_0\frac{\partial}{\partial t}\left(\frac{\rho}{\epsilon_0}\right)\Rightarrow\vec{\nabla}\cdot\vec{J}=-\frac{\partial\rho}{\partial t}$$

which says that charge is conserved.

These are the equations shown in the showcase in the Derby lobby.