Field of a magnetized object 6.2.1
This will follow your author's treatment very closely

For a single dipole, we have: $\vec{A}(\vec{r}_p)=\frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}_{ip}}{r_{ip}^3}$

Suppose the object though consists of a many dipoles. We can superimpose the vector potentials to obtain:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\oiiint\frac{\vec{M}(\vec{r}_i)\times\vec{r}_{ip}}{r_{ip}^3}d\tau$$

where the magnetization (M) is defined as the magnetic dipole moment per unit volume. As before:

$$\vec{\nabla}_i\left(\frac{1}{r_{ip}}\right)=\frac{\vec{r}_{ip}}{r_{ip}^3}$$

So:

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\oiiint\left[\vec{M}(\vec{r}_i)\times\left(\vec{\nabla}_i\frac{1}{r_{ip}}\right)\right]d\tau_i$$

This can be integrated by parts to give (ultimately):

$$\vec{A}(\vec{r}_p)=\frac{\mu_0}{4\pi}\oiiint\frac{1}{r_{ip}}\left[\vec{\nabla}_i\times\vec{M}(r_i)\right]d\tau_i+\frac{\mu_0}{4\pi}\oiint\frac{1}{r_{ip}}\left[\vec{M}(r_i)\times d\vec{a}_i\right]$$

The first term looks like the potential of a volume current: $\vec{J}_b=\vec{\nabla}\times\vec{M}$.
The second term looks like the potential of a surface current: $\vec{K}_b=\vec{M}\times\hat{n}$.

This means the potential due to a magnetization can be represented as bound volume and surface currents:

$$\vec{A}=\frac{\mu_0}{4\pi}\oiiint\frac{\vec{J}_b(r_i)}{r_{ip}}d\tau_i+\frac{\mu_0}{4\pi}\oiint\frac{\vec{K}_b(r_i)}{r_{ip}}da_i$$

Which you will note looks a whole lot like what we have for polarized objects:

$$\rho_b=-\vec{\nabla}\cdot\vec{P}:\sigma_b=\vec{P}\cdot\hat{n}$$

Example 6.1

What is the magnetic field of a uniformly magnetized sphere.

Let M be along the z-axis so that:

$$\vec{J}_b=\vec{\nabla}\times\vec{M}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & M\end{vmatrix}=0 \quad \text{(could have done this easier)}$$

$$\vec{K}_b = \vec{M} \times \hat{n} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & M \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{vmatrix} = \hat{x}(-M\sin\theta\sin\phi) - \hat{y}(-M\sin\theta\cos\phi)$$

$$\Rightarrow \vec{K}_b = M\sin\theta(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) = M\sin\theta\,\hat{\phi}$$

(of course, having it worked out makes this easier to see).

Now according to your author, this is equivalent to a spinning sphere of charge. Let's obtain the field in such a situation. This is Example 5.11

The sphere has a surface charge σ , of radius R and is spinning with ω. Let the angular velocity be tilted away from the polar angle by an angle Ψ in the x-z plane.

The vector potential is:

$$\vec{A} = \frac{\mu_0}{4\pi} \oiint \frac{\vec{K}}{r_{ip}} da$$

Here, $\vec{K} = \sigma\vec{v}$

Then

$$\vec{\omega} = \omega\sin\psi\,\hat{x} + 0\,\hat{y} + \omega\cos\psi\,\hat{z}$$

and we find the surface current by:

$$\vec{K} = \sigma\vec{v} = \sigma\vec{\omega} \times \vec{r}_i = \sigma\omega \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \sin\psi & 0 & \cos\psi \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{vmatrix}$$

$$= \sigma\omega(-\sin\theta\sin\phi\cos\psi\,\hat{x} - \hat{y}(\sin\psi\cos\theta - \sin\theta\cos\phi\cos\psi) + \hat{z}(\sin\psi\sin\theta\sin\phi))$$

Upon integration on the sphere, the only term that will survive the integration is:

$$-\sin\psi\cos\theta\,\hat{y}$$

So the vector potential will be given by:

$$\vec{A} = -\hat{y}\frac{\mu_0}{4\pi}\sigma\omega R^2(2\pi)\sin\psi \int_{\theta=0}^{\theta=\pi} \frac{\sin\theta\, d\theta\cos\theta}{\left[r_i^2 + r_p^2 - 2r_i r_p\cos\theta\right]^{1/2}}$$

Your author integrates this by a trig substitution: (see page 237)

$$u \equiv \cos\theta \Rightarrow \int_{-1}^{1} \frac{du}{\sqrt{R^2 + r_p^2 - 2Rr_p u}} = \frac{-1}{3R^2r^2}\left[(R^2 + r_p^2 + Rr_p)|R - r_p| - (R^2 + r_p^2 - Rr)(R + r)\right]$$

To give the result (letting the angle now be along z):

note that $\omega \times \hat{r}_p = \omega r_p \sin\phi\,\hat{\phi}$

$$\vec{A} = \frac{\mu_0\sigma R}{3}(\vec{\omega} \times \vec{r}_p) \quad \text{inside the sphere}$$

$$\vec{A} = \frac{\mu_0\sigma R^4}{3r_p^3}(\vec{\omega} \times \vec{r}_p) \quad \text{outside the sphere}$$

From here:

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{2}{3}\mu_0\sigma R\vec{\omega}$$