More on currents and the Biot-Savart law 2016

As written (Eq. 5.32), we have it it's pure form:

$$\vec{B}=\frac{\mu_0}{4\pi}\int\frac{\vec{I}\,x\,\hat{r}_{ip}}{r_{ip}^2}dL$$

Now it is pretty clear to me that L consists of locations strictly defined by the currents because if the current was not in a portion where L happened, in that region I would be zero and thus no contribution although maybe a wire did exist. The upshot here is this: L represents paths of currents. They also represent wires if current is traveling exactly along the wires.

Now let's consider just what is the meaning of $\vec{I}\,dL$ and how it would be written in different forms. One thing is for sure: we can superimpose magnetic fields. So suppose we had a series of lines of current, and maybe each still carrying a current I. Then to calculate the total magnetic field we would have to do something like this:

$$\vec{B}=\sum_{i=1}^{N}\vec{B}_i$$

Where B_i represents the magnetic field that came from the Ith current in the ith wire. About this we can certainly agree. If this became a continuum situation, then:

$$B=\int_{\text{all currents}} d\vec{B}$$

But now for each current, we have:

$$d\vec{B}=\frac{\mu_0}{4\pi}\int\frac{d\vec{I}\,x\,\vec{r}_{ip}}{r_{ip}^3}dL$$

So now, although for sure we were integrating over currents previously, we now have to regard the previous work as integrating over wires and the current I that was mentioned in fact is more of a current function (again not a density). It is pretty clear to me now that the currents that we are integrating over need to be perpendicular to the current directions, so maybe writing it like this: In particular let wires be directed along the x-direction and then lined up along the y-direction.

$$\vec{B}=\frac{\mu_0}{4\pi}\oiint\frac{\frac{d\vec{I}}{dy}\,x\,\vec{r}_{ip}}{r_{ip}^3}dxdy$$

Note that this is essentially now the same as the surface current definition, $\vec{K}=\frac{d\vec{I}}{dL_{\perp}}$ so $\vec{B}=\frac{\mu_0}{4\pi}\oiint\frac{\vec{K}\,x\,\vec{r}_{ip}}{r_{ip}^3}dA$ but now dA is understood to be the area over which the currents exist. Now let's suppose that we have a 3-d structure composed of these sheets of current which are stacked on top of each other. In the same way, you define a current density by: $\vec{J}=\frac{d\vec{K}}{dz}$ where z is the direction perpendicular to both y and x. Then just as before, the magnetic field from this would be given by:

$$\vec{B}=\frac{\mu_0}{4\pi}\oiiint\frac{\vec{J}\,x\,\vec{r}_{ip}}{r_{ip}^3}d\tau$$

where τ is now understood to be the volume over which current densities exist.