2.2.4 The curl of E

Stokes' theorem states:

$$\oiint_{surface} \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{A} = \oint_{path} \vec{E} \cdot d\vec{l}$$

Let's evaluate for a point charge at the origin the second part.

$$\vec{E} = k\frac{q}{r^2}\hat{r} : d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin(\theta)\,d\phi \Rightarrow \vec{E}\cdot d\vec{l} = k\frac{q}{r^2}dr$$

$$\int \vec{E}\cdot d\vec{l} = kq\int_a^b \frac{dr}{r^2} = -kq\left[\frac{1}{r}\right]_a^b = kq\left[\frac{1}{a} - \frac{1}{b}\right]$$

If, however we consider a closed path, then this is exactly zero. Thus,

$$\oint_{path} \vec{E}\cdot d\vec{l} = 0 \Rightarrow \oiint_{surface} \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{A} = 0 \Rightarrow \vec{\nabla} \times \vec{E} = 0$$

Which this means that electrostatic fields have divergence but not curl. Note though that we used here that this electrostatic field arose from charges, if the field arises from other entities, this is not necessarily true.