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k_1 1 → | 2 k_2

0

$$\tilde f_I = \tilde A_I e^{i(k_1 z - \omega t)}$$ Incid

$$\tilde f_R = \tilde A_R e^{i(-k_1 z - \omega t)}$$ Ref

$$\tilde f_T = \tilde A_T e^{i(k_2 z - \omega t)}$$

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$$\tilde f(z,t) = \begin{cases} \tilde f_I + \tilde f_R & (z<0) \\ \tilde f_T & (z>0) \end{cases}$$

$$\tilde f(0^-,t) = \tilde f(0^+,t)$$

$$\frac{\partial \tilde f}{\partial z}(0^-) = \frac{\partial \tilde f}{\partial z}(0^+)$$

$$\tilde{A}_I + \tilde{A}_R = \tilde{A}_T \; : \; k_1(\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$$

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)\tilde{A}_I \qquad v = \frac{\omega}{k}$$

$$\tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2}\right)\tilde{A}_I \qquad n = \frac{c}{v}$$

$$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)\tilde{A}_I \qquad \tilde{A}_T = \left(\frac{2v_2}{v_1 + v_2}\right)\tilde{A}_I$$

$$A_R e^{i\delta_R} \qquad A_I e^{i\delta_I} \qquad A_T e^{i\delta_T}$$

Strings $v = \sqrt{\frac{T}{\mu}}$

① ②

if $\mu_2 < \mu_1$ $\delta_R = \delta_L = \delta_T$ 0° shift

if $\mu_2 > \mu_1$ $\delta_R + \pi = \delta_L = \delta_T$ 180° shift

if $\mu_2 \to \infty$ $\delta_R + \pi = \delta_T$ 180° shift

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint\!\!\oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_C + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iiint (\vec{\nabla}\cdot\vec{E})\,d\tau$$

$$q_{enc} = \iiint \rho\,d\tau$$

$$\iiint \left[(\vec{\nabla} \cdot \vec{E}) - \frac{\rho}{\varepsilon_0} \right] d\tau = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oiint \vec{B} \cdot d\vec{A} = \iiint (\vec{\nabla} \cdot \vec{B}) \, d\tau$$

$$\Rightarrow \vec{\nabla} \cdot \vec{B} = 0$$

$$\oint_c \underline{\vec{E} \cdot d\vec{s}} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt} \oiint \vec{B} \cdot d\vec{A}$$

$$\iint_{\text{surf}} (\vec{\nabla} \times \vec{E}) \cdot d\vec{A} = \oint_c \vec{E} \cdot d\vec{s}$$

$$\oiint\left[(\vec{\nabla}\times\vec{E})+\frac{d\vec{B}}{dt}\right]\cdot d\vec{A}=0$$

Farady $$\vec{\nabla}\times\vec{E}=-\frac{d\vec{B}}{dt}$$

Amper $$\oint\vec{B}\cdot d\vec{s}=\mu_0 I_c+\mu_0\varepsilon_0\frac{\partial\phi_E}{\partial t}$$

$$\oint_c\vec{B}\cdot d\vec{s}=\oiint_s(\vec{\nabla}\times\vec{B})\cdot d\vec{A}$$

$$I_c=\oiint_s\vec{J}\cdot d\vec{A} \qquad \phi_E=\oiint_s\vec{E}\cdot d\vec{A}$$

$$\oiint_s(\vec{\nabla}\times\vec{B})\cdot d\vec{A}=\mu_0\oiint\vec{J}\cdot d\vec{A}$$

$$+\frac{d}{dt}\varepsilon_0\mu_0\oiint\vec{E}\cdot d\vec{A}$$

$$\oint\!\!\oint_S \left[(\vec{\nabla}\times\vec{B}) - \mu_0 \vec{J} - \varepsilon_0 \mu_0 \frac{d\vec{E}}{dt} \right]\cdot d\vec{A}$$

$$= 0$$

$$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

if $Q = 0$, $I_c = 0$

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \cancel{\vec{\nabla}\times\vec{B} =} \ \partial B$$

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \left(\begin{array}{l} \vec{\nabla}\times\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{B} = \varepsilon_0 \mu_0 \dfrac{\partial \vec{E}}{\partial t} \end{array} \right)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \underline{\vec{\nabla}(\vec{\nabla} \cdot \vec{E})} - \vec{\nabla}^2 \vec{E}$$

$$\vec{\nabla} \times \vec{E} = -\nabla^2 \vec{E}$$

$$-\underline{\vec{\nabla}^2 \vec{E}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B}$$

$$-\nabla^2 \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2 \vec{B} = -\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 \vec{E}\right) = \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\nabla \times E\right) = -\frac{\partial^2 B}{\partial t^2}$$

$$\frac{\partial B}{\partial t} = \frac{\partial^2 R}{\partial t^2}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$\vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

$$-\frac{\partial}{\partial t} \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow$$

$$\vec{\nabla}^2 \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}^2 \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ Faraday's

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 E$$

$$-\vec{\nabla}^2 \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}\times\vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$-\vec{\nabla}^2 E = -\frac{\partial}{\partial t}\left(\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla}^2 \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\left(\vec{\nabla}\times\vec{B}\right) = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)$$

$$= \varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right) - \vec{\nabla}^2 B$$

$$-\vec{\nabla}^2\vec{B} = -\varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t}$$

$$\vec{\nabla}^2\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 B}{\partial t^2} = 0$$

$$\boxed{\begin{aligned} \nabla^2\vec{E} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{aligned}}$$

$$v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\varepsilon_0\mu_0\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial^2}{\partial t^2}(\varepsilon_0\mu_0\vec{E})$$

$$= -\varepsilon_0\mu_0\frac{\partial^2 \vec{E}}{\partial t^2}$$

But

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\Rightarrow \quad -\nabla^2\vec{E} = -\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \quad \nabla^2\vec{E} - \varepsilon_0\mu_0\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \varepsilon_0\mu_0\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\cancel{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = -}\ \varepsilon_0\mu_0$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right) = \varepsilon_0\mu_0\left(-\frac{\partial^2\vec{B}}{\partial t^2}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{\nabla}^2\vec{B}$$

BAC CAB

$$-\vec{\nabla}^2\vec{B} = -\varepsilon_0\mu_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\vec{\nabla}^2\vec{B} - \varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$\vec{E}(z,t) = \vec{\tilde{E}}_0 e^{i(kz-\omega t)}$$

$$\vec{B}(z,t) = \vec{\tilde{B}}_0 e^{i(kz-\omega t)}$$

$$\vec{\nabla}^2 \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}^2 \vec{E} = \vec{\tilde{E}}_0 (ik\,ik) e^{i(kz-\omega t)}$$

$$= -k^2 \vec{\tilde{E}}_0 e^{i(kz-\omega t)}$$

$$= -k^2 \vec{\tilde{E}}$$

$$\frac{\partial^2 \vec{\tilde{E}}}{\partial t^2} = (-i\omega)(-i\omega) \vec{\tilde{E}}$$

$$\nabla^2 E - \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = 0$$

$$-k^2 \hat{E} + \underbrace{(-\omega^2)}_{\varepsilon_0 \mu_0} \hat{E} = 0$$

if $k^2 = \omega^2 \varepsilon_0 \mu_0$

$$\frac{k}{\omega} = \varepsilon_0 \mu_0$$

$$\frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$