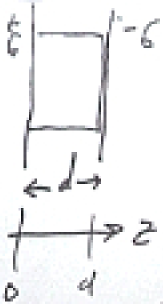
3.3

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$



$\vec{P} = g z \hat{z}$

$\vec{E} = ?$

$\vec{E}$ all q

$\vec{D}$ free q

$$\vec{E} = \frac{\vec{D} - \vec{P}}{\varepsilon_0} : \text{e.o.a.}$$

$E \quad \sigma_b,\ \rho_b,\ \sigma_f$

D: 

$\oint \vec{D}\cdot d\vec{A} = Q_{enc}$

$DA' = Q_{enc}$

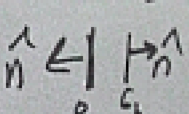
$Q_{enc} = \sigma_f A'$

$\vec{D} = \sigma_f \hat{z}$

if $\vec{P} = 0$ $\quad \vec{D} = \varepsilon_0 \vec{E}$, $\vec{E} = \frac{\sigma}{\varepsilon_0}$

$$\vec{E} = \frac{(\sigma - gz)\hat{z}}{\varepsilon_0}$$

σ_b : $\sigma_b = \hat{n}\cdot\vec{P}$ $\qquad \vec{P} = gz\hat{z}$



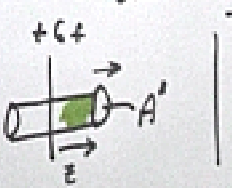
$\sigma_b = -0$

$\sigma_b = \hat{n}\cdot\vec{P} = gd$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\vec{\nabla} \rightarrow \frac{\partial}{\partial z}\hat{z}$$

$$\rho_b = -\frac{\partial}{\partial z}(gz) = -g$$

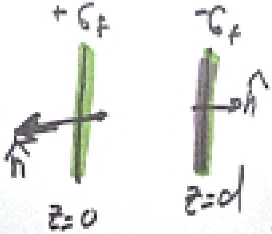


$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$EA' = \frac{1}{\varepsilon_0}\left(\sigma_f A' - g z A'\right)$$

$$\vec{E} = \frac{1}{\varepsilon_0}\left(A' - gz\right)\hat{z}$$

$$\vec{P} = (z+a)^2 \hat{z}$$



$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

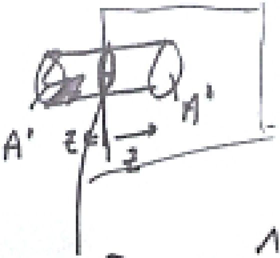
$$\vec{D} = \sigma_f \hat{z}$$

$$\frac{\vec{D} - \vec{P}}{\varepsilon_0} = \vec{E}$$

$$\vec{E} = \left[\frac{\sigma_f}{\varepsilon_0} - \frac{1}{\varepsilon_0}(z+a)^2\right]\hat{z}$$

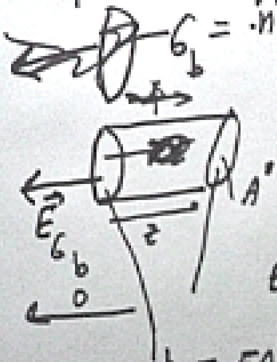
$$\vec{P} = (z+a)^2 \hat{z}$$

$$\sigma_b = \hat{n}\cdot\vec{P} = -(a)^2$$



$$\sigma_b = \hat{n}\cdot\hat{P} = -P\big|_{z=0}$$

$$= -0^2\hat{z}$$



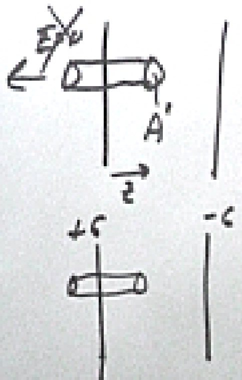
$$\rho_b = -2(z+a)$$

$$\phi_E = EA' = \frac{Q_{enc}}{\epsilon_0}$$

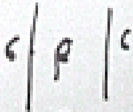
$$Q_{enc} = \sigma_b\big|_{z=0} A' + \int_0^z A'\,(-2(z+a))\,dz$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -2(z+a)$$

$$\sigma_b = \hat{n}\cdot\vec{P} = (d+a)^2$$

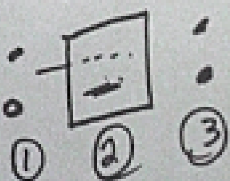


$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$



$$\vec{E} = \frac{\sigma_f}{\varepsilon_0}\hat{z}$$

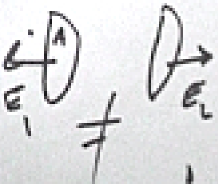
σ_f



$$Q_{enc} = \sigma_b A' + A' \cdot (-2)\int_0^z (z+a)\,dz$$

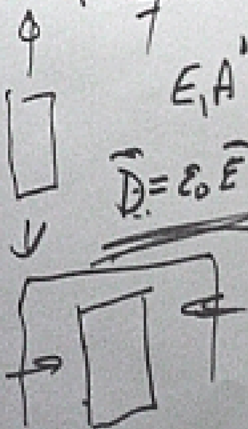
$z=0$

$$= \sigma_b A' - 2A'\left(\frac{z^2}{2} + az\right)$$

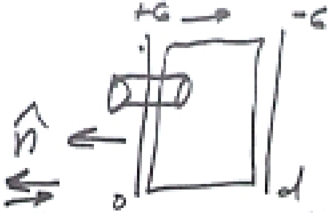


$$E_1 A' + E_2 A' = \frac{Q_{enc}}{\varepsilon_0}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$



Coulomb's LAW



$$\vec{P} = \epsilon_0 \chi_E \vec{E}$$

$$\sigma_b = \hat{n} \cdot \vec{P}$$

$z=0$

$$\oint \vec{D} \cdot d\vec{A} = Q_{f\,enc}$$

$$DA' = \sigma_f A'$$

$$\vec{D} = \sigma_f \hat{z}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_E \vec{E}$$

$$= \epsilon_0 (1 + \chi_E) \vec{E}$$

$$\vec{E} = \frac{\sigma_f}{\varepsilon_0(1+\chi_e)}\hat{z}$$

$$\vec{P} = \varepsilon_0\chi_e E = \frac{\varepsilon_0\chi_e}{\varepsilon_0(1+\chi_e)}\sigma_f\hat{z}$$

$$\vec{P} = r^2\hat{r}$$

$$\mathrm{I}: \vec{D}(4\pi r^2) = \sigma_f$$

$$\vec{D} = \frac{\sigma_f}{4\pi r^2}\hat{r}$$

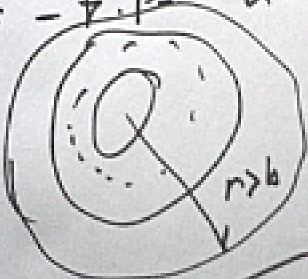
$$\underset{@a}{\sigma_b} = -a^2$$

$$@b \ \underset{@b}{\sigma_b} = b^2$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

$$\vec{\nabla} = \frac{\partial}{\partial r}\,\hat{r}$$

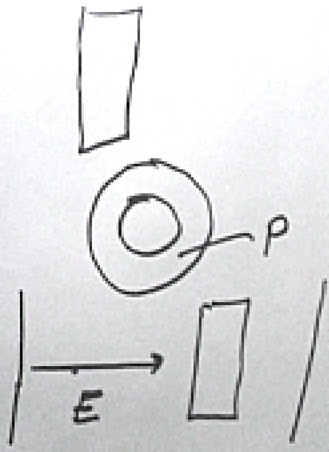
$$\rho_b = -\vec{\nabla}\cdot\vec{P} = -2r$$ Between a and b



$$\vec{E}(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$$

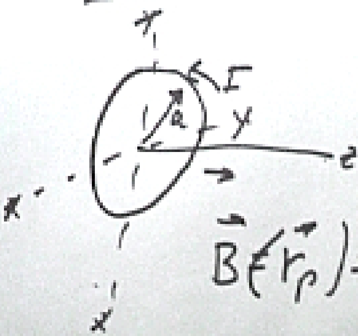
$$\vec{E} = \frac{Q_{enc}}{\varepsilon_0}$$

$$Q_{enc} = \sigma_{b\,@\,a}\cdot 4\pi a^2 + 4\pi\int_{r=a}^{r=b}\rho\, r^2\, dr + \sigma_{b\,@\,b}\cdot 4\pi b^2$$

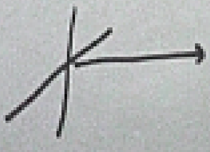


P
E

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_C$$



$$\vec{B}(\vec{r}_p) = \frac{\mu_0 I}{4\pi}\int_{curr} \frac{d\vec{l}_i \times \vec{r}_{ip}}{|\vec{r}_{ip}|^3}$$



$$\vec{r}_i = x_i \cdot \hat{x} + y_i \cdot \hat{y}$$

$$\vec{r}_p = z_p \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = z_p \hat{z} - x_i \cdot \hat{x} - y_i \cdot \hat{y}$$

$d\vec{L}_i \times \vec{r}_{ip}$

$\vec{L}_i = \cancel{a}\ a\cos\theta\hat{x} + a\sin\theta\hat{y}$

$d\vec{L}_i = -a\sin\theta\hat{x} + a\cos\theta\hat{y}$

$$\int_{\theta=0}^{\theta=2\pi}$$

$$d\vec{L}_i \times \vec{r}_{ip} = a\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -x_i & -y_i & z_p \end{vmatrix}$$

$$a\left[\hat{x}\, z_p \cos\theta - \hat{y}\left(-\sin\theta\, z_p\right) + \hat{z}\left(y_i \cdot \sin\theta + x_i \cdot \cos\theta\right)\right]$$

$$y_i = a\cos\theta$$

$$x_i = a\sin\theta$$

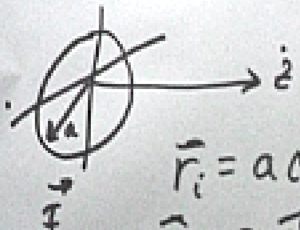
$$a^2 \hat{z}\left[2\sin\theta\cos\theta\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} \frac{2a^2 \sin\theta\cos\theta}{\left[a^2+z_p^2\right]^{3/2}}\,\hat{z}$$

$$\cancel{\frac{\mu_0 I \quad 2}{4\pi \quad a^2 +}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\cdot 2a^2}{[a^2+z_p^2]^{3/2}} \hat{z} \left[-\frac{1}{2} \cos^2(\theta) \right]_0^{2\pi}$$

$$\cancel{\Rightarrow \vec{B}} = 0$$



$$\vec{r}_i = a\cos\theta\,\hat{x} + a\sin\theta\,\hat{y}$$

$$\vec{r}_p = z_p\,\hat{z}$$

$$\vec{r}_{ip} = -a\cos\theta\,\hat{x} - a\sin\theta\,\hat{y} + z_p\hat{z}$$

$$\vec{L} \times \vec{r}_i = a\cos\theta\,\hat{x} + a\sin\theta\,\hat{y}$$

$$d\vec{L} = -a\sin\theta\,\hat{x} + a\cos\theta\,\hat{y}$$

$$a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -a\cos\theta & -a\sin\theta & z_p \end{vmatrix}$$

$$a^2 (\sin^2\theta + \cos^2\theta) = a^2$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{z_p a^2}{[a^2+z_p^2]^{3/2}} \, a \, d\theta \, \hat{z}$$

$$= \frac{\mu_0 I}{4\pi} \frac{z_p a^2}{[a^2+z_p^2]^{3/2}} \cdot a \cdot 2\pi \hat{z}$$