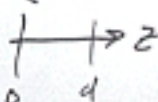


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$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$



$$\vec{E} = \frac{\vec{D} - \vec{P}}{\epsilon_0} \quad \text{c.o.v.}$$


$$\vec{P} = q z \hat{z}$$

$$\vec{E} = ?$$

$$\vec{E} \text{ all } q$$

$$\vec{D} \text{ free } q$$

$$E \quad \sigma_b, \rho_b, \sigma_f$$

D:  $\oint \vec{D} \cdot d\vec{A} = Q_{enc}$
 $DA' = Q_{enc}$

$$Q_{enc} = \sigma_f A'$$

if $\vec{P} = 0$ $\vec{D} = \epsilon_0 \vec{E}$, $\vec{E} = \frac{\sigma}{\epsilon_0}$

$$\vec{E} = \frac{(\sigma - \rho z) \hat{z}}{\epsilon_0}$$

$\sigma_b : \sigma_b = \hat{n} \cdot \vec{P}$ $\vec{P} = \rho z \hat{z}$

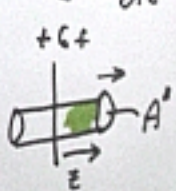
$\hat{n} \leftarrow | \rightarrow \hat{n}$ $\sigma_b = -\rho$

$\sigma_b = \hat{n} \cdot \vec{P} = \rho d$

$$P_b = -\vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} \rightarrow \frac{\partial}{\partial z} \hat{z}$$

$$P_b = -\frac{\partial}{\partial z} (\rho z) = -\rho$$



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\epsilon A' = \frac{1}{\epsilon_0} (CA' - \rho z A')$$

$$\vec{E} = \frac{1}{\epsilon_0} (A' - \rho z) \hat{z}$$



$$\vec{P} = (z+a) \hat{z}$$

A vertical green rod is shown at $z=a$. It has a negative charge density $-G_f$. A normal vector \hat{n} points to the right.

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

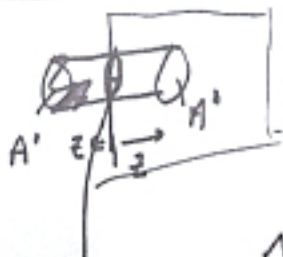
$$\vec{D} = G_f \hat{z}$$

$$\frac{\vec{D} - \vec{P}}{\epsilon_0} = \vec{E}$$

$$\vec{E} = \left[\frac{G_f}{\epsilon_0} - \frac{1}{\epsilon_0} (z+a) \right] \hat{z}$$

$$\vec{P} = (z+a) \hat{z}$$

$$G_b = \hat{n} \cdot \vec{P} = -(a)$$



$$\sigma_b = \hat{n} \cdot \hat{p} = -P|_{z=0} = -\sigma_b z$$

A diagram of a cross-section of the shell. A normal vector \hat{n} points to the right, and a pressure vector \hat{p} points to the left. The stress σ_b is indicated as acting on the surface.

$$P_b = -2(z+ra)$$

A diagram showing a cylindrical shell with a cross-section labeled A' . A horizontal axis labeled z points to the right. The shell is shown in perspective, with the front and back edges visible. A pressure P_b is indicated acting on the shell.

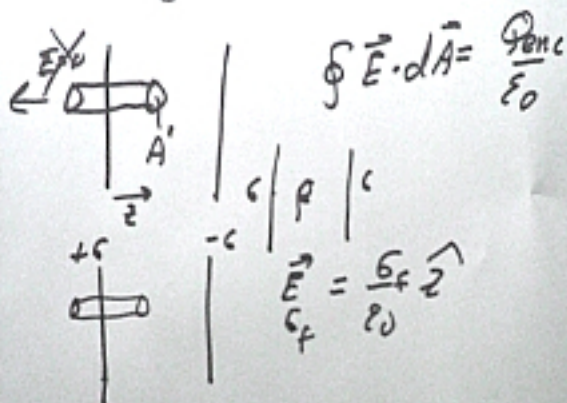
$$\phi_E = EA' = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \int_0^z \sigma_b A' dz = \int_0^z (-2(z+ra)) dz$$

~~EA'~~

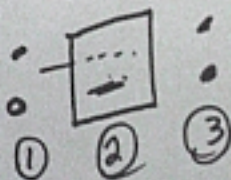
$$P_b = -\vec{\nabla} \cdot \vec{P} = -2(z+a)$$

$$G_b^* = \hat{n} \cdot \vec{P} = (d+a)$$



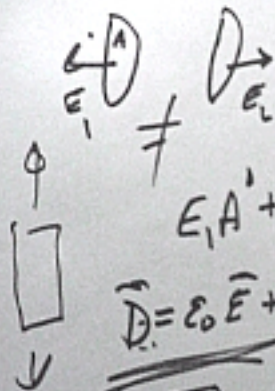
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma_f}{\epsilon_0} \hat{z}$$



$$Q_{enc} = \int_{z=0}^z \epsilon_0 A' + A' \cdot (-2) \int_0^z (z+1) dz$$

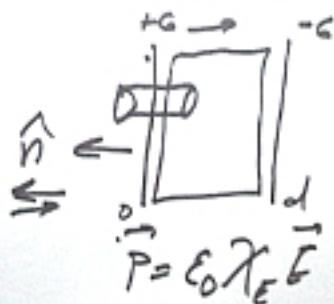
$$= \epsilon_0 A' - 2A' \left(\frac{z^2}{2} + Gz \right)$$



$$\epsilon_1 A' + \epsilon_2 A' = \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Coulomb's
LAW



$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$G_b = \oint_{z=0} \vec{n} \cdot \vec{P}$$

$$\oint \vec{D} \cdot d\vec{A} = Q_{enc}$$

$$DA' = G_f A'$$

$$\vec{D} = G_f \hat{z}$$

$$\begin{aligned} \vec{D} &= \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} \\ &= \epsilon_0 (1 + \chi_e) \vec{E} \end{aligned}$$

$$\vec{E} = \frac{Q_f}{\epsilon_0(1+\chi_e)} \hat{r}$$

$$\vec{P} = \epsilon_0 \chi_e E = \frac{\epsilon_0 \chi_e Q_f}{\epsilon_0(1+\chi_e)} \hat{r}$$



$$\vec{P} = r^2 \hat{r}$$

$$I: \oint \vec{D} \cdot d\vec{A} = Q_f$$

$$\vec{D} = \frac{Q_f}{4\pi r^2} \hat{r}$$

$$\left. \begin{array}{l} \hat{r} \\ \leftarrow \end{array} \right) \begin{array}{l} \oint_a \vec{D} \cdot d\vec{A} = -a^2 \\ \oint_b \vec{D} \cdot d\vec{A} = b^2 \end{array}$$

$$\oint_b \vec{D} \cdot d\vec{A} = b^2$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$\vec{\nabla} = \frac{\partial}{\partial r} \hat{r}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -2r \quad \text{Between } a \text{ and } b$$



$$\vec{E}(4\pi r^2)$$

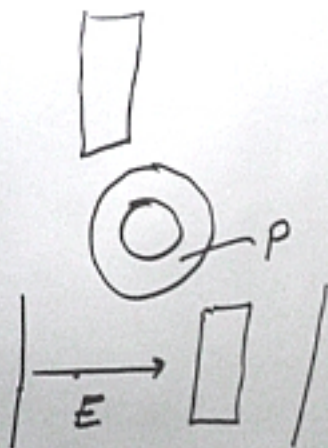
$$= \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{E} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = Q_b \cdot 4\pi a^2$$

$$+ 4\pi \int_{r=0}^{r=b} \rho r dr$$

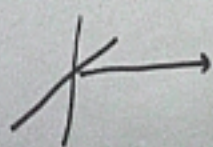
$$+ Q_b \cdot 4\pi b^2$$



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_c$$



$$\vec{B}(\vec{r}_p) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}_i \times \vec{r}_{ip}}{|\vec{r}_{ip}|^3}$$



$$\vec{r}_i = x_i \hat{x} + y_i \hat{y}$$

$$\vec{r}_p = z_p \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = z_p \hat{z} - x_i \hat{x} - y_i \hat{y}$$

$$d\vec{L}_i \times \vec{r}_{ip}$$

$$\vec{L}_i = a \cos\theta \hat{x} + a \sin\theta \hat{y}$$

$$d\vec{L}_i = -a \sin\theta \hat{x} + a \cos\theta \hat{y}$$

$$\theta = a\pi$$

$$\int_{\theta=0}$$

$$d\vec{L}_i \times \vec{r}_{ip} = a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -x_i & -y_i & z_p \end{vmatrix}$$

$$a \left[\hat{x} z_p \cos \theta \right. \\ \left. - y (-\sin \theta z_p) \right]$$

$$+ \hat{z} (y_i \sin \theta + x_i \cos \theta)$$

$$y_i = a \cos \theta$$

$$x_i = a \sin \theta$$

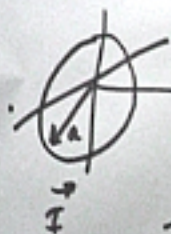
$$a^2 \hat{z} \int_{2\pi} [2 \sin \theta \cos \theta]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{\theta=0}^{\theta=2\pi} \frac{2a^2 \sin \theta \cos \theta}{[a^2 + z_p^2]^{3/2}} \hat{z}$$

$$\frac{\mu_0 I a^2}{4\pi [a^2 + z_p^2]}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{2Q^2}{[a^2+z^2]^{3/2}} \hat{z} \left[\begin{matrix} 1 \\ -2 \\ 0 \end{matrix} \cos(\theta) \right]^{2\pi}$$

$$\vec{B} = 0$$



$$\vec{r}_i = a \cos\theta \hat{x} + a \sin\theta \hat{y}$$

$$\vec{r}_p = z \hat{z}$$

$$\vec{r}_{ip} = -a \cos\theta \hat{x} - a \sin\theta \hat{y} + z \hat{z}$$

$$d\vec{L} = a \cos\theta \hat{x} + a \sin\theta \hat{y}$$

$$d\vec{L} = -a \sin\theta \hat{x} + a \cos\theta \hat{y}$$

$$a \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -a \cos\theta & -a \sin\theta & z_p \end{vmatrix}$$

$$a^2 (\sin^2\theta + \cos^2\theta) = a^2$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{z_p a^2 a d\theta \hat{z}}{[a^2 + z_p^2]^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi} \frac{z_p a^2}{[a^2 + z_p^2]^{3/2}} \cdot a \cdot 2\pi \hat{z}$$