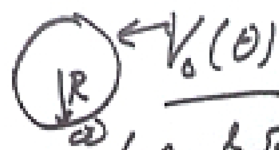
382

Ex 3.6



$$V=\sum_{l=0}^{\infty}\left(A_l r^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Insid $r<R \Rightarrow B_l=0$

$$\sum A_l r^l P_l(\cos\theta)$$

$$V_0(\theta)=\sum A_l R^l P_l(\cos\theta)$$

$V_0(\theta) = k\sin^2\left(\frac{\theta}{2}\right)$

write as $P_\ell(\cos\theta)$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos\theta)$

$P_0 \quad P_1$

<u>outside</u>

$\infty \Rightarrow A_\ell \to 0$

$$V(r,\theta) = \sum_{\ell=0}^{\infty} \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

@ $r=R$, $V=V_0(\theta)$

$$\int_0^{\pi} V_0(\theta)\, P_{\ell}' \sin\theta\, d\theta$$

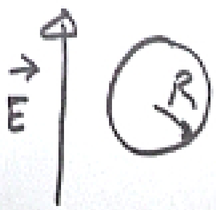
$$=$$

$$\frac{B_{\ell}'}{R^{\ell+1}} \frac{\int_0^{\pi} P_{\ell'}^2 \sin\theta\, d\theta}{L}$$

$$\frac{B_{\ell}'}{R^{\ell+1}} \cdot \frac{2}{2\ell+1}$$

$$\underline{B_{\ell}} = \frac{2\ell+1}{2} \cdot R^{\ell+1} \int_0^{\pi} V_0(\theta) P_{\ell} \sin\theta\, d\theta$$

Ex 3.8



$$\vec{E} = -\frac{dV}{dz}\hat{z}$$

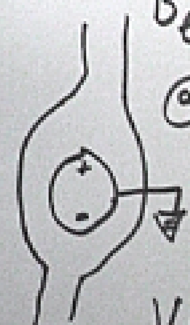
$$V = -Ez + c$$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

$@ r = R, \; V = 0$

$$A_\ell R^\ell + \frac{B_\ell}{R^{\ell+1}} = 0$$

$$B_\ell = A_2 R^{2\ell+1}$$



$@ r = R, \; \theta = 0$

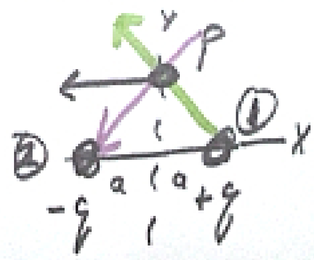
$V = 0 \quad \ell = 0, \ell = 1$

$$V(r,\theta) = A_\ell \left(r - \frac{R^3}{r^2} \right) \cos\theta$$

$z = r\cos\theta$

$\boxed{\vec{E} = -\vec{\nabla} V}$ $\quad A_\ell = -E_0$

$$V(r,\theta) = -E_0 \left(r^1 - \frac{R^3}{r^2} \right) \cos\theta$$



$$\vec{E}_p = \sum_{i=1}^{n} k \frac{q_i}{r_{ip}^2} \hat{r}_{ip}$$

$$\vec{r}_1 = a\hat{x} + 0\hat{y}$$

$$\vec{r}_2 = -a\hat{x} + 0\hat{y}$$

$$\vec{r}_p = 0\hat{x} + y_p\hat{y}$$

$$\vec{r}_{1p} = \vec{r}_p - \vec{r}_1 = -a\hat{x} + y_p\hat{y}$$

$$\vec{r}_{2p} = \vec{r}_p - \vec{r}_2 = a\hat{x} + y_p\hat{y}$$

$$\vec{E}_p = kq\left[\frac{+(-a\hat{x} + y_p\hat{y})}{[a^2 + y_p^2]^{3/2}} - \frac{(a\hat{x} + y_p\hat{y})}{[a^2 + y_p^2]^{3/2}}\right]$$

$$\vec{E}_p = \frac{-2akq}{[a^2 + y_p^2]^{3/2}}\hat{x}$$

Close to x: $\ominus \leftarrow \oplus$

$y_p \to 0$; $y_p = 0$

$$\vec{E}_p = \frac{-2k\cancel{a}q}{a^{3/2}}\hat{x}$$

$y_p \to$ Large:

$$(a^2+y_p^2)^{3/2} = y_p^3\underbrace{\left(\left(\frac{a}{y_p}\right)^2+1\right)^{3/2}}_{1}$$

$$\vec{E}_p = \underline{\frac{-2akq}{y_p^3}}(1)\hat{x}$$

$$\vec{E}_p = -\frac{2akq\hat{x}}{y_p^3\left[1+\frac{a^2}{y_p^2}\right]^{3/2}}$$

$$\vec{E}_p = \frac{-2akq\hat{x}}{y_p^3}\left(1 - \frac{3\left(\frac{a}{y_p}\right)^2}{2} + \frac{15\left(\frac{a}{y_p}\right)^4}{8} \vdots\right)$$

Small $\vec{p}$ $\vec{P} = \sum_{j=1}^{2} q_j \vec{r}_j$

Dipole moment $\vec{p} = \sum_{q_i} q_i \vec{r}_i$

$P =$ Polarization

$\vec{P} = \frac{\overrightarrow{\text{dipole moment}}}{\text{volume}}$

$\vec{p} = C\,m$ $\vec{P} = \frac{C\,m}{m^3} = \frac{C}{m^2}$

$$\vec{r}_1 = a\hat{x} + 0\hat{y} \quad (+q)$$

$$\vec{r}_2 = -a\hat{x} + 0\hat{y} \quad (-q)$$

$$\vec{p} = (+q)(a\hat{x} + 0\hat{y})$$

$$+ (-q)(-a\hat{x} + 0\hat{y})$$

$$= qa \cdot 2\hat{x} = 2aq\hat{x}$$

$$\boxed{\begin{matrix} \vec{p} \\ \vec{p} \end{matrix}}$$

$$\rho(\vec{r}_i)$$

$$\vec{p} = \int_{\substack{\text{all} \\ V}} \vec{r}_i \left(\rho(\vec{r}_i)\right) d^3 r_i$$

$$\vec{r}_p = x_p\hat{x} + y_p\hat{y}$$

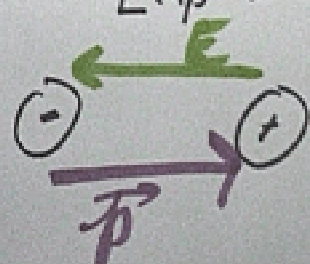
$$\vec{r}_1 = a\hat{x} + 0\hat{y}$$

$$\vec{r}_2 = -a\hat{x} + 0\hat{y}$$

$$\vec{r}_{1p} = (x_p - a)\hat{x} + y_p\hat{y}$$

$$\vec{r}_{2p} = (x_p + a)\hat{x} + y_p\hat{y}$$

$$\hat{E}_p = kq \left[\frac{(x_p - a)\hat{x} + y_p\hat{y}}{\left[(x_p - a)^2 + y_p^2\right]^{3/2}} + (-)\frac{(x_p + a)\hat{x} + y_p\hat{y}}{\left[(x_p + a)^2 + y_p^2\right]^{3/2}} \right]$$



$$\vec{p} = \sum q_i \cdot \vec{r}_i$$

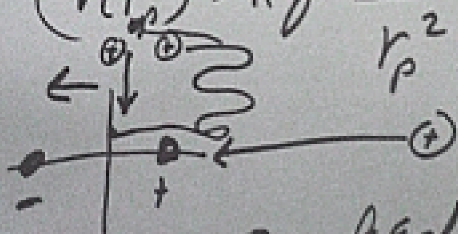
$\vec{r}_{+p} = (x_p - \frac{d}{2})\hat{x} + y_p\hat{y}$

$\vec{r}_{-p} = (x_p + \frac{d}{2})\hat{x} + y_p\hat{y}$

$$|\vec{r}_{+p}| = \sqrt{r_p^2 + \frac{d^2}{4} - 2\vec{r}_p \cdot \vec{r}_+}$$

$$|\vec{r}_{-p}| = \sqrt{r_p^2 + \frac{d^2}{4} + 2\vec{r}_p \cdot \vec{r}_-}$$

$$V(\vec{r}_p) = kq\,\frac{d\cos\theta}{r_p^2}$$



Sarob Aliu $V(\vec{r}_p) = \frac{kqd}{x_p^2}$

$$W = Q\,\frac{kqd}{x_p^2}$$