382

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$V = X\,Y\,Z$$

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} = 0$$

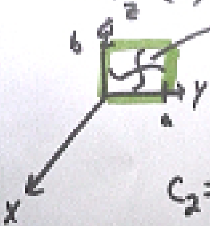
$$\div\ XYZ$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} + \frac{1}{Z}\frac{d^2 Z}{dz^2} = 0$$

$$C_1 \qquad C_2 \qquad C_3$$

$C_1 + C_2 + C_3 = 0$

∿ z (−)



@ $x=0$, $V = V_0(y,z)$

err

$C_2 = -k^2$ $C_3 = -p^2$

$C_1 = k^2 + p^2$

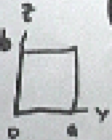
$V=0$ @ $x=a$

$$V = XYZ$$

$$V(x,y,z) = \left(Ae^{+\sqrt{k^2+p^2}} + Be^{-\sqrt{k^2+p^2}}\right)$$

$$\left(C\sin(ky) + D\cos(ky)\right)$$

$$\left(E\sin(pz) + F\cos(pz)\right)$$



@ $z=0 \Rightarrow F=0$

@ $y=0 \Rightarrow D=0$

@ $x=\infty$, $V \to 0$

$\Rightarrow A=0$

$$V(x,y,z) = A e^{-\sqrt{k^2+p^2}} \sin(ky)$$

$$\sin(pz)$$

@ $y = a \quad V = 0 \Rightarrow ka = n\pi$

$n = 1, 2, \ldots$

$$k = \frac{n\pi}{a}$$

@ $z = b \quad V = 0 \Rightarrow pb = m\pi$

$$p = \frac{m\pi}{b} \quad m = 1, 2, \ldots$$

A_{nm}

$$V(x,y,z) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} A_{nm} \times$$

$$e^{-\left[n^2 \frac{\pi^2}{a^2} + m^2 \frac{\pi^2}{b^2}\right]^{1/2} x}$$

$$\sin\left(\frac{n\pi}{a} y\right)$$

$$\sin\left(\frac{m\pi}{b} z\right)$$

$$@\, x = 0 : \; V = V_0(x, y)$$

$$\int_{z=0}^{z=b} \int_{y=0}^{y=a} \circledast \; \sin\left(\frac{n'\pi}{a} y\right) \sin\left(\frac{m'\pi}{b} z\right) dy\, dz$$

$$\int_{z=0}^{b} \sin\left(\frac{m\pi z}{b}\right) dz$$

$\underline{\text{design}}$

$$-\frac{b}{m\pi}\cos\left(\frac{m\pi z}{b}\right)\Bigg|_0^b$$

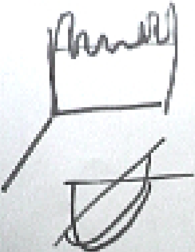
$m=1 \quad = -\left[\frac{b}{\pi}(-1) - \frac{b}{\pi} 1\right]$ odd

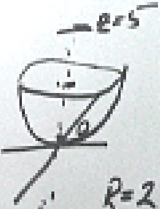
$m=2 \quad -\left[\frac{b}{2\pi}(1) - \frac{b}{2\pi}\right] = 0$

$$A_{nm} = \frac{16 V_0}{\pi n m} \quad \text{(n and m odd)}$$

$$= 0 \quad \text{(n or m even)}$$

$e^{-\pi\sqrt{\quad}x}$



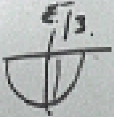


$R=2$

$z \stackrel{?}{=} 2-5$

$2+5$

$=5-2=3$



$$V(x,y,z)$$

$$= \frac{16 V_0}{\pi} \sum_{n=1,3,5}^{\infty} \sum_{m=1,3,5}^{\infty} \Bigg[e^{-\pi \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2}\, x}$$

$$(\text{times}) \quad \frac{\sin\left(\frac{n\pi y}{a}\right)}{n} \; \frac{\sin\left(\frac{m\pi z}{b}\right)}{m} \Bigg]$$

ϕ $V(\phi)$ const

$V(r,\theta)$

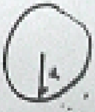
$\vec{\nabla}^2 V = 0$

$$\vec{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$+R$

$l(l+1) \quad -\Theta$

$$R(r) = Ar^{l} + \frac{B}{r^{l+1}}$$



Inside
$B=0$

outside
$A=0 \quad \left(\begin{matrix} V \to 0 \\ \text{at } \infty \end{matrix} \right)$

$$\Theta(\theta) = P_\ell(\cos\theta)$$

$x = \cos\theta$

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$\vdots$$

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta)$$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l$$

$$\text{Inside} \Rightarrow B_l = 0$$

$$V_{\text{inside}}(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

$V(R,\theta) = V_0(\theta)$

$$V_0(\theta) = \sum_{\ell=0}^{\infty} A_\ell R^\ell P_\ell(\cos\theta)$$

$$\times \int_{\theta=0}^{\pi} P_{\ell'}(\cos\theta) d \quad [\sin\theta d\theta]$$

$$\frac{2}{2\ell'+1}\delta_{\ell\ell'}$$

$$= \frac{2}{2\ell'+1} A_{\ell'} R^{\ell'}$$

$$\frac{2}{2l'+1} A_{l'} R^{l'} = \int_{\theta=0}^{\pi} V_0(\theta) P_{l'}(\cos\theta)\sin\theta\, d\theta$$

Eyeball method $V_0(\theta) = k \sin^2\left(\frac{\theta}{2}\right)$

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}\left(1 - \cos\theta\right)$$

$$V = k\frac{1}{2} P_0(x) - \frac{1}{2} P_1(x)$$

2 terms: $A_0 = \frac{k}{2R^0} = \frac{k}{2}$

$$A_1 = -\frac{k}{2R}$$

$z = r\cos\theta$

$$V(r,\theta) = \frac{k}{2}\left(1 - \frac{z}{R}\right)$$

Do outside

Soln next time