(382)

$$V(x,y) = [A\sin(kx) + B\cos(kx)] \times [Ce^{ky} + De^{-ky}]$$

① $x=0,\ V=(0,y) \Rightarrow B=0$

② $x=a,\ V=0,y$

$ka = n\pi,\ n=1,2,3\ldots$

$$V(x,y) = \sum A_n \sin\left(n\pi\frac{x}{a}\right)$$

$$[Ce^{ky} + De^{-ky}]$$

$$\cdot\!-\!- \quad \underline{C\sinh(ky) + D\cosh(ky)}$$

$$\therefore D = 0$$

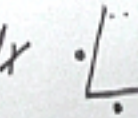
$$V(x,y) = \sum A_n \sin\left(n\pi\frac{x}{a}\right)\sinh($$

$$\sin\left(\frac{n\pi}{a}y\right)$$

$$\frac{n\pi}{a}$$

$$V(x,y) = \sum_{n=1}^{\infty} A_n \sin\left(n\pi \frac{x}{a}\right) \sinh\left(n\pi \frac{y}{a}\right)$$

$$\int_{x=0}^{x=a} x \sin\left(m\pi \frac{x}{a}\right) dx$$



at $y = b$, $V = V_0$

$$\int_{x=0}^{x=a} V_0 \sin\left(m\pi \frac{x}{a}\right) dx$$

$$= \int_{x=0}^{x=a} A_m \sin^2\left(m\pi \frac{x}{a}\right) \sinh\left(m\pi \frac{b}{a}\right) dx$$

$$= A_m \sinh\left(m\pi \frac{b}{a}\right) \int_0^a \sin^2\left(m\pi \frac{x}{a}\right) dx$$

$$\int_0^a \sin^2\left(m\pi\frac{x}{a}\right)dx = \frac{x}{2}\Big|_0^a$$

$$= \frac{a}{2}\left|\int_0^a \sin\left(m\pi\frac{x}{a}\right)dx\right.$$

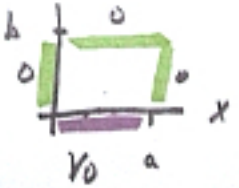
$$= \frac{-\cos\left(m\pi\frac{x}{a}\right)}{m\pi/a}\Big|_0^a$$

$$\frac{-\cos(m\pi)}{m\pi/a} - \frac{1}{m\pi/a}$$

$$-\frac{2a}{m\pi}\begin{cases} m = \text{even} & 0 \\ m = \text{odd} & \end{cases}$$

$$A_m = \frac{\left[\text{SVG} \right]}{\sinh\left(m\pi\frac{b}{a}\right)\left(\frac{a}{2}\right)}$$

$$V(x,y) = \sum_{m=2,4,6\ldots} \frac{-\left(\frac{2q}{m\pi}\right)}{\sinh\left(m\pi\frac{b}{a}\right)\left(\frac{a}{2}\right)} \left(\sin\left(m\pi\frac{x}{a}\right) \sinh\left(\frac{m\pi}{b}y\right) \right)$$



$$V(x,y) = [A\sin(kx) + B\cos(kx)]$$

$$[Ce^{-ky} + De^{+ky}]$$

$$k = \frac{n\pi}{a}$$

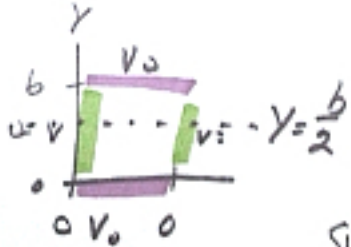
@ $y = b$, $V = 0$

$$Ce^{-kb} + De^{+kb} = 0$$

$$D = -\frac{Ce^{-ikb}}{e^{+ikb}} = -Ce^{-2ikb}$$

$$\left[Ce^{-iky} + De^{+iky}\right]\ [\ ]$$

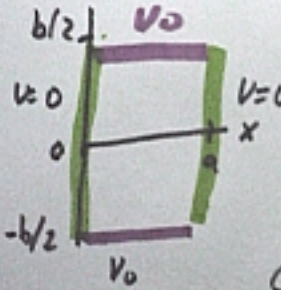
$$\vec{E} = -\vec{\nabla}V$$



$$y' = y - \frac{b}{2}$$

$$\sinh \propto e^{ky} - e^{-ky}$$

$$\cosh \propto e^{ky} + e^{-ky}$$



$$fun(-y) = fun(y)$$

$$\sinh(-y) = -\sinh(y)$$

$$\cosh(-y) = \cosh(y)$$

$$V_n(x,y) = A_n \sin\left(\frac{n\pi}{a}x\right) *$$

Y actual $\cosh\left(\frac{n\pi}{a}y\right)$

$$Ce^{-ky} + De^{+ky} = Y$$

$$Y(-y) = Y(+y)$$

$$\Rightarrow C = D$$

$$Y = \cancel{C} \cosh(ky)$$

eval with $y = \frac{b}{2}$

$V(x,y) = \sum_{m=\text{even}}$

@ $y = \frac{b}{2}$:

$V_0 = V(x, \frac{b}{2}) = \sum A_n \sin\left(\frac{n\pi x}{a}\right) \cosh\left(\frac{n\pi b}{2a}\right)$

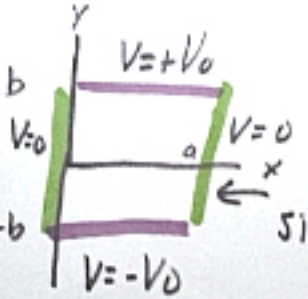
$$A_m = \frac{\int V_0 \sin\left(\frac{m\pi x}{a}\right) dx}{\frac{a}{2} \cosh\left(\frac{m\pi b}{2a}\right)}$$

m even

mode = 0

$\sum A_m \sin \cosh$

m even



$\sin\left(\frac{n\pi x}{a}\right)$

$Ce^{-ky} + De^{+ky} = 0$

$V(-y) = V(y)$

$\Rightarrow C = -D$

$Y: \sinh\left(\frac{n\pi y}{a}\right)$

$$V(x,y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) \times \sin\left(\frac{n\pi y}{a}\right)$$

② $y = b$: $V(x,b) = V_0$

$$V(x,b) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) \sinh\left(\frac{n\pi b}{a}\right)$$

$$\int_{x=0}^{x=a} V_0 \cdot \sin\left(\frac{m\pi x}{a}\right) dx$$

$$= A_m \sinh\left(\frac{m\pi}{a} y\right) \times \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx$$

$y=b$ $V=V_0$

$y=-b$ $V=-V_0$

$$Y = [Ce^{-ky} + De^{+ky}]$$

$$Y(-y) = -Y(y)$$

$$\Rightarrow C = -D$$

$$Y = C[-e^{-ky} + e^{+ky}]$$

$$\propto \sinh(ky)$$

$$\int_{x=0}^{x=a} V_0 \cdot \sin\left(\frac{m\pi x}{a}\right) dx$$

$$= A_m \sinh\left(\frac{m\pi}{a} y\right) \times \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx$$

$$\Downarrow$$

$$\frac{a}{2}, \; m = n$$

$$0, \; m \neq n$$

$$\int_0^a V_0 \sin\left(\frac{m\pi x}{a}\right) dx$$

$$= V_0 \left(-\frac{\cos\left(\frac{m\pi x}{a}\right)}{\frac{m\pi}{a}} \right)_0^a$$

$$-\frac{\cos(m\pi)}{\frac{m\pi}{a}} - 1$$

m even: $\frac{-2}{m\pi/a}$

m odd: 0

$$V_0\left(\frac{-2}{m\pi/a}\right) = \sum A_m \cdot \frac{a}{2} \cdot \sinh\left(\frac{m\pi b}{a}\right)$$

$$A_m = \frac{\frac{-2V_0}{m\pi/a}}{\frac{a}{2}\sinh\left(\frac{m\pi b}{a}\right)}$$

$$V(x,y) = \sum_{m \text{ even}} A_m \sinh\left(\frac{m\pi y}{a}\right) \sin\left(\frac{m\pi x}{a}\right)$$

$$\vec{E} = -\vec{\nabla} V$$

assume

$$V = -\frac{\frac{V_0 a}{\pi}}{\frac{a}{2}\sin\left(\frac{2\pi b}{a}\right)} \sinh\left(\frac{2\pi y}{a}\right) + \sin\left(\frac{2\pi x}{a}\right)$$

$$\vec{\nabla} V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y}$$

$$-\left[[\cdot] \frac{2\pi}{a} \cos\left(\frac{2\pi x}{a}\right)\hat{x} \right.$$

$$\left. [\cdot] \frac{2\pi}{a} \cosh\left(\frac{2\pi y}{a}\right)\hat{y} \right]$$

$$V(x,y) = \sum_{m=1}^{\infty} A_m \sin\left(\frac{m\pi x}{a}\right) e^{-\frac{m\pi y}{a}}$$

evaluate @ $y=0$

① $y=0$

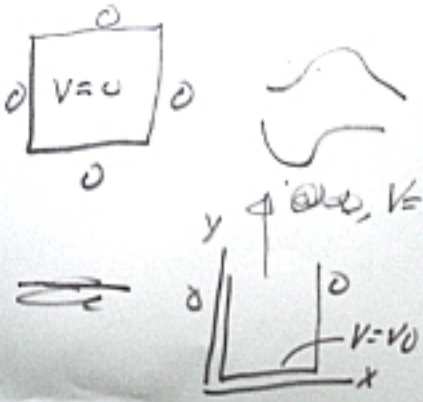
$$V(x,0) = \sum A_m \sin\left(\frac{m\pi x}{a}\right) \cdot 1$$

$$\times \int_{x=0}^{x=a} (\quad) \sin\left(\frac{n\pi x}{a}\right) dx$$

$$\frac{-2}{n\pi/a} = A_n \frac{a}{2}$$

0 mod

$$V(x,y) = \sum_{n \text{ even}} \frac{2}{n\pi/a} \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{n\pi y}{a}}$$



$$[Ce^{-ky} + De^{+ky}]$$

Require @ $y=\infty$, $V=0$

$$Y = Ce^{-ky}$$