33)

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Esfera

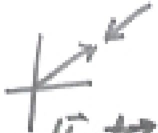
$$\Rightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$V = -\int_{\infty}^{r} \vec{E}\cdot dr\,\hat{r}$$

$$V = \int_{\infty}^{r} \vec{E}\cdot d\vec{r}$$

$d\hat{r} \to d\vec{l} = dr\,\hat{r}$

$\pm r\,d\theta\,\hat{\theta}$



$\pm r\sin\theta\,d\phi\,\hat{\phi}$

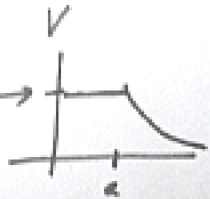
$d\vec{l} = \cancel{d\vec{r}} = -dr\,\hat{r}$

$$V = \int_{\infty}^{r} \vec{E}\cdot d\vec{l}$$

$$= -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0}\frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

$k = \frac{1}{4\pi\epsilon_0}$

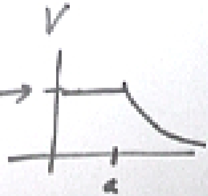
$V = \frac{kQ}{a} \rightarrow$ 

$\vec{E} = -\vec{\nabla} V$

$\vec{dl} = -dr\,\hat{r}$

$k = \frac{1}{4\pi\varepsilon_0}$

$V = \frac{kQ}{a} \rightarrow$



$\vec{E} = -\vec{\nabla} V$

$\vec{dl} = -dr\,\hat{r}$

$$\vec{E} = -\vec{\nabla} V \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = -\vec{\nabla}\cdot(\vec{\nabla} V)$$

$$\frac{\rho}{\varepsilon_0} = -\vec{\nabla}^2 V$$

$\vec{\nabla}^2$ Laplacian

$$\nabla_\rho^2 \rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)$$

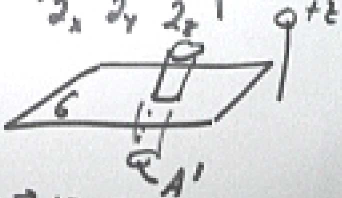
$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \quad \text{Poisson's}$$

$$\nabla^2 V = 0$$

$$\vec{\nabla} \times \vec{E} = -\vec{\nabla} \times (\vec{\nabla} V)$$

$$\vec{\nabla} \times (\vec{\nabla} V) = 0$$

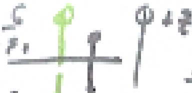
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \end{vmatrix} = 0$$



$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A') = \frac{\sigma A'}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \begin{cases} +\hat{z}, & z> \\ -\hat{z}, & z<0 \end{cases}$$

$E=\frac{\sigma}{2\varepsilon_0}$



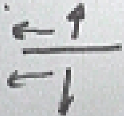
$E=-\frac{\sigma}{2\varepsilon_0}$

$$\Delta\vec{E}=\frac{\sigma}{2\varepsilon_0}\hat{z}-\frac{-\sigma}{2\varepsilon_0}\hat{z}$$

Δ = final − initial

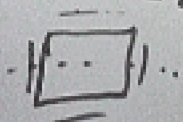
$$\Delta\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{z}$$

$$\cancel{\Delta\vec{E}_{\parallel}}=$$



$$\Delta E_{\perp}=\frac{\sigma}{\varepsilon_0}$$

$$\Delta E_{\parallel}=0$$

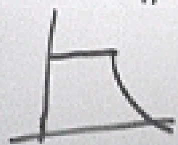


$$\vec{E}_{Above}-\vec{E}_{Below}=\frac{\sigma}{\varepsilon_0}\hat{n}$$

$$V_A - V_B = -\oint \vec{E}\cdot d\vec{s}$$

$$\Rightarrow \Delta V = 0$$

$$\vec{\nabla} V_A - \vec{\nabla} V_B = -\frac{\sigma}{\epsilon_0}\hat{n}$$



$$\frac{\partial V}{\partial n} \equiv (\vec{\nabla} V)\cdot\hat{n}$$

$$\frac{\partial V_A}{\partial n} - \frac{\partial V_B}{\partial n} = -\frac{\sigma}{\epsilon_0}$$

q_1 @ $\vec{r}_1$ $\qquad$ q_2 @ $\vec{r}_2$

$$V = \frac{W}{q_0} \Rightarrow qV$$

$$V_p = \frac{q_1}{4\pi\varepsilon_0 r_{1p}} \qquad \frac{q_1}{4\pi\varepsilon_0 r_{12}}$$

$$r_{12} = |\vec{r}_{12}| = |\vec{r}_{21}|$$

$$W = W_1 + W_{21} + W_{31} + W_{32}$$

$$= 0 + q_2 \frac{q_1}{4\pi\varepsilon_0 r_{12}} + q_3 \frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_3 q_2}{4\pi\varepsilon_0 r_{23}}$$

$$\iiint\limits_{\text{all } q_i} \rho_i V_i \, d^3 r_i$$

$$W = \frac{1}{2} \iint \sigma_i V_i \, dA_i$$

$$W = \frac{1}{2} \oint \lambda_i V_i \, ds$$

$$W = \frac{1}{2} \iiint\limits_{\text{all } q_i} \rho_i V_i \, d^3 r_i$$

$$W=\frac{1}{2}\iiint\rho_i\,V_i\,d^3r_i$$

all q_i

$$W=\frac{1}{2}\oiint\sigma_i\,V_i\,dA_i$$

$$W=\frac{1}{2}\oint\lambda_i\,V_i\,ds$$

$$W=\frac{1}{2}\oiiint\rho_i\,V_i\,d^3r_i$$

all q_i

$$\vec{\nabla}_p\cdot\vec{E}_p=\frac{\rho_i}{\varepsilon_0}$$

$$W=\frac{1}{2}\oiiint(\varepsilon_0\vec{\nabla}_p\cdot\vec{E}_p)V_p\,d^3r$$

$$\vec{\nabla}_p \cdot \vec{E}_p = \frac{\rho}{\varepsilon_0} \qquad \vec{E} = -\vec{\nabla} V$$

$$W = \frac{1}{2} \iiint \left(\varepsilon_0 \vec{\nabla}_p \cdot \vec{E}_p \right) V_p \, d^3 r$$

$$\int \frac{d}{dx}(fg) = \int f \frac{dg}{dx} \, dx + \int g \left(\frac{df}{dx} \right) dx$$

$$\cancel{W = \frac{1}{2} \varepsilon_0 \Big[- \iiint \vec{E} \cdot \vec{\nabla} V}$$

$$W = \frac{1}{2} \varepsilon \Big[- \iiint \vec{E} \cdot \vec{\nabla} V \, d^3 r + \oiint V \vec{E} \cdot d\vec{A} \Big]$$

$$\int \frac{d}{dx}(fg) = \int f \frac{dg}{dx}\,dx + \int g\left(\frac{df}{dx}\right)dx$$

$$W = \frac{1}{2}\varepsilon_0 \left[\cancel{-\iiint \vec{E}\cdot\nabla V} \right.$$

$$r'^3$$

$$\vec{r}_{i,p} = \vec{r}_p - \vec{r}_i$$

$$d^3 r_p \xrightarrow[(-)]{} d^3 r_i$$

$$W = \frac{1}{2}\varepsilon_0 \left[\boxed{\iiint \vec{E}\cdot\vec{E}\,d^3r} + \boxed{\oiint V\vec{E}\cdot d\vec{A}} \right]$$

$$0 \xrightarrow[\infty]{}$$

$$W = \frac{1}{2}\varepsilon_0 \iiint \vec{E}\cdot\vec{E}\,d^3r$$

$(-)$

$$W = \frac{1}{2}\varepsilon_0 \left[\boxed{\iiint \vec{E}\cdot\vec{E}\, d^3r} + \boxed{\oint V \vec{E}\cdot d\vec{A}} \right]$$

$$0 \xrightarrow[\infty]{}$$

$$W = \frac{1}{2}\varepsilon_0 \iiint \vec{E}\cdot\vec{E}\, d^3r$$

$$W = \iiint u_E \, d^3r$$

$$u_E = \frac{1}{2}\varepsilon_0 (\vec{E}\cdot\vec{E})$$

wed. Ex 2.8