332

σ

$$dq_i = \sigma s_i\, ds_i\, d\phi_i$$

$$\vec{r}_i = x_i \hat{x} + y_i \hat{y}$$

$$\vec{r}_p = \quad z_p \hat{z}$$

$$\vec{r}_{ip} = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E}_p = k\sigma \int_0^{2\pi} d\phi_i \int_b^a \left[\frac{s_i\, ds_i \left(-x_i \hat{x} - y_i \hat{y} + z_p \hat{z}\right)}{\left[s_i^2 + z_p^2\right]^{3/2}} \right]$$

$$\vec{E}_p = k\,\sigma \cdot 2\pi\, z_p \hat{z}$$

$$\int_a^b \frac{s_i\,ds_i}{\left[s_i^2 + z_p^2\right]^{3/2}}$$

$$= -k\,\sigma\, 2\pi\, z_p \hat{z} \left[\frac{1}{\sqrt{b^2+z_p^2}} - \frac{1}{\sqrt{a^2+z_p^2}}\right]$$

$$\left[\frac{1}{\sqrt{b^2+z_p^2}} - \frac{1}{\sqrt{a^2+z_p^2}}\right]$$

$$= k\,\sigma\, 2\pi\, z_p \hat{z} \left[\frac{1}{\sqrt{a^2+z_p^2}} - \frac{1}{\sqrt{b^2+z_p^2}}\right]$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\oiiint \rho\, d^3r$$

$$\oiint \vec{E}\cdot d\vec{A} = \oiiint (\vec{\nabla}\cdot\vec{E})\, d^3r$$

$$\oiiint (\vec{\nabla}\cdot\vec{E})\, d^3r = \frac{1}{\varepsilon_0}\oiiint \rho\, d^3r$$

$$\oiiint \left[\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\varepsilon_0}\right] d^3r = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$$

$\vec{E} \downarrow \rho$ $\qquad$ $\rho \downarrow$ Coulomb's

GAUSS $\oiint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\iiint \rho\, d^3r$

GAUSS's theorem

$$\iiint (\vec{\nabla}\cdot\vec{E})\, d^3r = \oiint \vec{E}\cdot d\vec{A}$$

$$\iiint (\vec{\nabla}\cdot\vec{E})\, d^3r = \iiint \frac{\rho}{\varepsilon_0}\, d^3r$$

$$\iiint \left[\vec{\nabla}\cdot\vec{E} - \frac{\rho}{\varepsilon_0}\right] d^3r = 0$$

$$\therefore\ \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}_p\cdot\vec{E}_p = \frac{\rho_{(i)}}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Leftarrow \rho \cdot 4\pi \delta^3(r)$$

$$\vec{\nabla}_p \cdot \frac{\hat{r}_{ip}}{r_{ip}^2} = 4\pi \delta^3(r)$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \vec{\nabla}_p \cdot \frac{\hat{r}_{ip}}{r_{ip}^2}$$

$$\boxed{\vec{E} = \frac{q}{\varepsilon_0 (4\pi)} \frac{\hat{r}_{ip}}{r_{ip}^2}}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \longleftarrow \rho \cdot 4\pi\,\delta^3(r)$$

$$\vec{\nabla}_p \cdot \frac{\hat{r}_{ip}}{r_{ip}^2} = 4\pi\,\delta^3(r)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\,\vec{\nabla}_p \cdot \frac{\hat{r}_{ip}}{r_{ip}^2}$$

$$\boxed{\vec{E} = \frac{q}{\varepsilon_0 (4\pi)} \frac{\hat{r}_{ip}}{r_{ip}^2}}$$

$$\oiint (\vec\nabla \times \vec{E}) \cdot d\vec{A} = \oint \vec{E} \cdot d\vec{l}$$

Surface

typsh $\vec{E} = \frac{kq}{r^2}\hat{r}$. $d\vec{l} = dr\hat{r} + r d\theta\hat\theta + r\sin\theta\, d\phi\hat\phi$

$$\oint \vec{E} \cdot d\vec{l} = \oint \frac{kq}{r^2} dr$$

$$\int_a^b \vec{E} \cdot d\vec{l} = kq\int_a^b \frac{dr}{r^2} = kq\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$\vec\nabla \times \vec{E} = \vec{0}$$

$$= kq\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$\vec{E} = k\left[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}\right]$$

$$\vec{\nabla}\times\vec{E} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{bmatrix}$$

$$= \left[\frac{\partial}{\partial y}(\underset{0}{3xz}) - \frac{\partial}{\partial z}(2yz)\right]\hat{x}$$

$$= \underline{-2y\hat{x}}$$

$$\hat{x}\left(\frac{\partial}{\partial y}\underset{0}{(3xz)} - \frac{\partial}{\partial z}(\overset{2y}{2yz})\right)$$

$$\left[\frac{\partial}{\partial y}(3xz) - \frac{\partial}{\partial z}(2yz)\right]\hat{x}$$

$$= -2y\hat{x}$$

$$\hat{x}\left(\frac{\partial}{\partial y}\underbrace{(3xz)}_{0} - \frac{\partial}{\partial z}\overbrace{(2yz)}^{2y}\right)$$

$$-\hat{y}\left(\frac{\partial}{\partial x}(3xz) - \frac{\partial}{\partial z}\underbrace{(xy)}_{0}\right)$$

$$-\hat{y}(3z)$$

$$+\hat{z}\left(\frac{\partial}{\partial x}\underbrace{(2yz)}_{0} - \frac{\partial}{\partial y}(xy)\right)$$

$$=\hat{z}(-x)$$

$$\vec{\nabla}\times\vec{E} = -2y\hat{x} - 3z\hat{y} - x\hat{z}$$

Since $\vec{\nabla}\times\vec{E} \neq 0$

$\Rightarrow \vec{E}$ not electrostatic field

$$\vec{E} = k\left[y^2\hat{x} + (2xy+z^2)\hat{y} + 2yz\hat{z}\right]$$

$$\vec{\nabla}\times\vec{E} = k\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cancel{y^2} & \cancel{2xy+z^2} & \cancel{2yz} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy+z^2) & 2yz \end{vmatrix}$$

$$\hat{x}\,(2z-2z)=0$$

$$-\hat{y}\,(0-0)+\hat{z}\,(2y-2y)=0$$

$$\vec{\nabla}\times\vec{E}=\vec{0}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{E} = k\left[0x + 2x + 2y\right] = \frac{\quad}{\varepsilon_0}$$

$$= \frac{2k}{\varepsilon_0}[x+y]$$

$$k \neq \cancel{c} \; \frac{1}{4\pi\varepsilon_0}$$

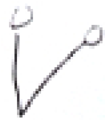
$$\vec{E} \quad \frac{N}{C} \qquad = k \; m^2$$

$$k = \frac{N}{C m^2}$$

$$E = \frac{kQ}{r^2} \Rightarrow k = \frac{N}{C}\cdot\frac{m^2}{C}$$

$$-\vec{\nabla} V = \vec{E}$$

$$\boxed{\vec{\nabla} V = -\vec{E}}$$



$$V = -\int_{\text{Ref}}^{r_p} \vec{E} \cdot d\vec{l}$$

$$\vec{\nabla} \cdot \left(\vec{\nabla} V = -\vec{E} \right)$$

$$\vec{\nabla}^2 V = -\vec{\nabla} \cdot \vec{E} = -\frac{\rho}{\varepsilon_0}$$

$\vec{\nabla} \cdot \vec{\nabla}$

$$\vec{\nabla}^2 V = -\rho/\varepsilon_0$$

charge free $\Rightarrow$ $\underline{\vec{\nabla}^2 V = 0}$

$$\begin{tikzcd}
 & V \arrow[dl, bend right, "-\vec{\nabla} V"'] \arrow[d, shift left] \\
E \arrow[ur, "\int E\,dl"'] \arrow[r, "\vec{\nabla}\cdot\vec{E}"'] & \rho \arrow[u, shift left] \arrow[l, bend left, "\text{coulomb's}"]
\end{tikzcd}$$

$\int E\,dl$

$$\vec{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$$