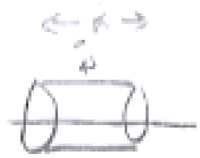


λ

GS, Radius s
centered on $\mathcal{L}$
On GS: E unif.
$\vec{E} \parallel \hat{s} \parallel d\vec{A}$

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \oint\!\!\oint E\,dA = E \oint\!\!\oint dA$$
$$= E(2\pi s h)$$

$$q_{enc} = \lambda h$$

$$E(2\pi s h) = \frac{\lambda h}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\,\hat{s}}$$

$\bullet\, y_p$

$\lambda \hat{z}$ E at ⊥ bisector

$$\vec{r}_i = z_i \hat{z}$$

$$\vec{r}_p = y_p \hat{y}$$

$$\vec{r}_{ip} = y_p \hat{y} - z_i \hat{z}$$

$$dq_i = \lambda\, dz_i$$

$$\vec{E} = k\lambda \int_{z=-\infty}^{z=+\infty} \frac{y_p \hat{y} - z_i \hat{z}}{[y_p^2 + z_i^2]^{3/2}}\, dz_i$$

$$= \cancel{k\lambda y_p \int}\; k\lambda y_p \hat{y} \int \frac{dz_i}{[y_p^2 + z_i^2]^{3/2}}$$

$$= \left.\frac{z_i}{y_p^2\sqrt{z_i^2 + y_p^2}}\right|_{-\infty}^{+\infty}$$

$$\div z_i = \left.\frac{k(\operatorname{sign} z_i)}{y_p^2\sqrt{1 + \frac{y_p^2}{z_i^2}}}\right|_{-\infty}^{+\infty} \quad \operatorname{Sign}(z_i)$$

$$= \frac{1}{y_p^2} - \frac{-1}{y_p^2}$$

$$\vec{E} = \frac{2}{y_p^2}\, k\lambda y_p \hat{y}$$

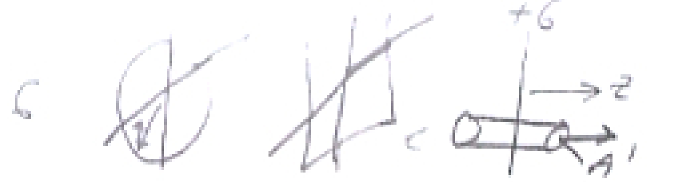
$$\underline{\frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}}$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$E = \cancel{2}\,\frac{1}{2\pi\varepsilon_0}\cdot\frac{\lambda}{y_p}\hat{y}$$

$$y_p \to s \quad \hat{y} = \hat{s}$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$$

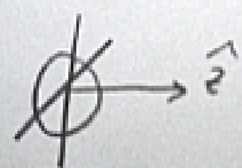


$V_a \to \infty$

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$2EA' = \frac{\sigma}{\varepsilon_0} A'$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{z}$$



$$\vec{r}_i = x_i\hat{x} + y_i\hat{y} + 0\hat{z}$$

$$\vec{r}_p = 0\hat{x} + 0\hat{y} + z_p\hat{z}$$

$$\vec{r}_{ip} = -x_i\hat{x} - y_i\hat{y} + z_p\hat{z}$$

$$dq = \sigma\,dx\,dy = \sigma\, s\,ds\,d\phi$$

$$\vec{E} = k\sigma \int_{\phi=0}^{2\pi} d\phi \int_{s=0}^{s=a} s\,ds\, \frac{-x_i\hat{x} - y_i\hat{y} + z_p\hat{z}}{\left[s^2 + z_p^2\right]^{3/2}}$$

$$= 2\pi k\sigma \int_0^a s\,ds\, \frac{z_p\hat{z}}{\left[s^2 + z_p^2\right]^{3/2}}$$

$$\vec{E} = 2\pi k\sigma \int z_p \hat{z} \int_0^a \frac{s\,ds}{[s^2+z_p^2]^{3/2}}$$

$$\cancel{= 2\pi k\sigma z_p \hat{z} \left(\frac{1}{\sqrt{x^2+z_p^2}} \right.}$$

$$\vec{E} = 2\pi k\sigma z_p \hat{z} \left[\frac{1}{\sqrt{s^2+z_p^2}} \right]_{0=s}^{a=s}$$

$$= 2\pi k\sigma z_p \hat{z} \left[\frac{-1}{|z_p|} + \frac{1}{\sqrt{a^2+z_p^2}} \right]$$

$$= 2\pi k\sigma \hat{z} \left[-1 + \frac{1}{\sqrt{1+\frac{a^2}{z_p^2}}} \right]$$

?

$$\vec{r}_i = x_i\hat{x} + y_i\hat{y} + 0\hat{z}$$

$$\vec{r}_p = 0\hat{x} + 0\hat{y} + z_p\hat{z}$$

$$\vec{r}_{ip} = -x_i\hat{x} - y_i\hat{y} + z_p\hat{z}$$

$$dq = \sigma\, dx\, dy = \sigma\, s\, ds\, d\phi$$

$$\vec{E} = k\sigma \int_{\phi=0}^{2\pi} d\phi \int_{s=0}^{s=a} s\, ds \, \frac{-x_i\hat{x} - y_i\hat{y} + z_p\hat{z}}{\left[s^2 + z_p^2\right]^{3/2}}$$

$$= 2\pi k\sigma \int_0^a s\, ds \, \frac{z_p\hat{z}}{\left[s^2 + z_p^2\right]^{3/2}}$$

Keep $z_p > 0$

$$\vec{E} = 2\pi k \sigma z_p \hat{z} \int_0^a \frac{s\,ds}{\left[s^2+z_p^2\right]^{3/2}}$$

$$= -2\pi k \sigma z_p \hat{z} \left[\frac{1}{\sqrt{s^2+z_p^2}}\right]_0^a$$

$$= -2\pi k \sigma \hat{z} \left[\frac{z_p}{\sqrt{s^2+z_p^2}}\right]_0^a$$

$$= -2\pi k \sigma \hat{z} \left[\frac{1}{\sqrt{1+s^2/z_p^2}}\right]_0^a$$

$$= -2\pi k \sigma \hat{z} \left[\frac{1}{\sqrt{1+\frac{a^2}{z_p^2}}} - 1\right]$$

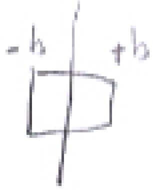
$$\lim_{a\to\infty} = 0$$

$$\vec{E} = 2\pi k \sigma \hat{z}$$

Gauss: $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$ $k = \frac{1}{4\pi\varepsilon_0}$

$$\vec{E} = 2\pi\left(\frac{1}{4\pi\varepsilon_0}\right)\sigma\hat{z}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$$



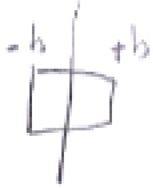
$\hat{E}$: $\vec{r}_i = x_i\hat{x} + y_i\hat{y} + z_i\hat{z}$

$$\vec{r}_p = z_p\hat{z}$$

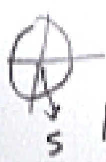
$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i\hat{x} - y_i\hat{y} + (z_p - z_i)\hat{z}$$

$$dq_i = \rho\, dz_i\, s_i\, ds_i\, d\phi_i$$

$$\vec{E}_p = \rho \cdot 2\pi \int_{s=0}^{s=a} s\,ds \int_{z_i=-b}^{z_i=b} dz \frac{(z_p - z_i)\hat{z}}{\left[s_i^2 + (z_p - z_i)^2\right]^{3/2}}$$



$\hat{E}$: $\vec{r}_i = x_i\hat{x} + y_i\hat{y} + z_i\hat{z}$



$$\vec{r}_p = z_p\hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i\hat{x} - y_i\hat{y} + (z_p - z_i)\hat{z}$$

$$dq_i = \rho\, dz_i\, s_i\, ds_i\, d\phi_i$$

$$\vec{E}_p = \rho \cdot 2\pi \int_{s=0}^{s=a} s\,ds \int_{z_i=-b}^{z_i=+b} dz\, \frac{(z_p - z_i)\hat{z}}{\left[s_i^2 + (z_p - z_i)^2\right]^{3/2}}$$

$$\vec{r}_i = x_i\hat{x} + y_i\hat{y} + z_i\hat{z}$$

$$\vec{r}_p = z_p\hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i\hat{x} - y_i\hat{y} + (z_p - z_i)\hat{z}$$

$$dq_i = \rho\, dz_i\, s_i\, ds_i\, d\phi_i$$

$$\vec{E}_p = \rho \cdot 2\pi h \int_{s=0}^{s=a} s\,ds \int_{z_i=-b}^{z_i=+b} dz \frac{(z_p - z_i)\hat{z}}{\left[s_i^2 + (z_p - z_i)^2\right]^{3/2}}$$

$$\vec{E}_p = 2\pi\rho h \int_{z_i=-h}^{z_i=+h} dz \int_{s=0}^{s=a} \frac{(z_p - z_i)\hat{z}\, s\,ds}{\left[s^2 + (z_p - z_i)^2\right]^{3/2}}$$

$$= 2\pi\rho h \int_{z_i=-b}^{+b} dz_i\, (z_p - z_i) \left[\frac{-1}{\sqrt{s^2 + (z_p - z_i)^2}}\right]_0^a$$

$$\lim_{a \to \infty} [\,] = 0 - \frac{-1}{\sqrt{(z_p - z_i)^2}}$$

$$= 2\pi\rho h \int_{z_i=-b}^{z_i=+b} dz_i \frac{(z_p - z_i)}{\sqrt{(z_p - z_i)^2}}$$

$[s^2+(z_p-z_i)]^{1/2}$

$$2\pi\rho\lambda \int dz_i\,(z_p-z_i)\left[\frac{-1}{\sqrt{s^2+(z_p-z_i)^2}}\right]_0^a$$

$z_i=-b$

0

$$\lim_{a\to\infty}[\,] = 0 - \frac{-1}{\sqrt{(z_p-z_i)^2}}$$

$$= 2\pi\rho\lambda \int_{z_i=-b}^{z_i=+b} dz_i \frac{(z_p-z_i)}{\sqrt{(z_p-z_i)^2}}$$

(1) $z_p > z_i$

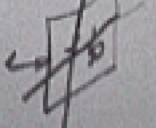
$$= k\,2\pi\rho\hat{z}\int_{-b}^{+b}\frac{dz_i}{1} = \frac{k\,2\pi\rho\hat{z}(2b)}{4\pi\varepsilon_0}$$

$$= \frac{\rho b}{\varepsilon_0}$$

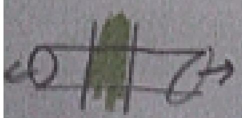
$$= 4\pi\frac{1}{4\pi\varepsilon_0}\,b\rho\hat{z} = b\rho\hat{z}$$

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

-b



$$\oint\!\!\oint \vec{E}\,d\vec{A} = E(2A')$$



$$q_{enc} = \rho(\cancel{2\pi}\,A') \quad z > z_i$$

$\cdot\, 2b$

$$= 2\pi\rho h \int_{z_1=-b} dz_1 \frac{z_p - z_1}{|(z_p - z_1)|^2}$$

(1) $z_p > z_1$

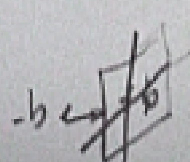
$$= k 2\pi\rho \hat{z} \int_{-b}^{+b} \frac{dz_1}{1} = k 2\pi\rho \hat{z}(2b)$$

$$= \frac{\rho b}{\varepsilon_0}$$

$$\vec{E} = 2\pi k\rho(2b)\hat{z}$$

$$= 4\pi \frac{1}{4\pi\varepsilon_0} b\rho\hat{z} = b\rho\hat{z}$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$



$$\oiint \vec{E}\, d\vec{A} = E(2A')$$

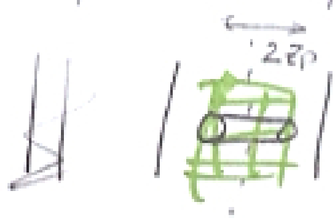
$$q_{enc} = \rho(A' \cdot 2b) \quad z > z_1$$

$$E(2A') = \frac{\rho A' \cdot 2b}{\varepsilon_0}$$

ρb

$p > 0$

$z_p < b$



$$\Phi_E = 2EA''$$

$$Q_{enc} = \rho \cdot A' \cdot 2z_p$$

$$2EA' = \frac{2z_p \rho A'}{\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{2z_p \rho}{\varepsilon_0} \hat{z}$$

$$\Phi_E = 2EA'$$

$$Q_{enc} = \rho \cdot A' \cdot 2z_p$$

$$2EA' = \frac{2z_p \rho A'}{\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{2z_p \rho}{\varepsilon_0}\hat{z} :$$

$$\vec{E} = 2\pi k \rho \hat{z} \int_{z_i=-b}^{z_i=+b} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_p - z_i)^2}}$$

$$= 2\pi k \rho \hat{z} \int_{z_i=-b}^{z_i=z_p} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_p - z_i)^2}} \quad 1$$

$$+ 2\pi k \rho \hat{z} \int_{z_i=z_p}^{z_i=b} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_i - z_p)^2}} \quad -1$$

[illegible]

$$2EA = \frac{2z_p \rho A}{\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{2z_p}{\varepsilon_0}\rho\hat{z}.$$

$$\vec{E} = 2\pi k \rho \hat{z} \int_{z_i=-b}^{z_i=+b} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_p - z_i)^2}}$$

$$= 2\pi k \rho \hat{z} \int_{z_i=-b}^{z_i=z_p} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_p - z_i)}} \; 1$$

$$+ 2\pi k \rho \hat{z} \int_{z_i=z_p}^{z_i=b} dz_i \, \frac{(z_p - z_i)}{\sqrt{(z_i - z_p)}} \; -1$$

$$= 2\pi k \rho \hat{z} \left[(z_p - -b) + (-1)(b - z_p) \right]$$