

3)

$$\oint \vec{E} \cdot d\vec{A} = \frac{\iiint \rho d\tau}{\epsilon_0}$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2}$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \nabla \cdot \hat{r})$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2}$$

$$\rho = \vec{\nabla} \cdot \vec{E}$$

$$\iiint \vec{\nabla} \cdot \frac{\hat{r}}{r^2} d\tau = \iint \frac{\hat{r}}{r^2} \cdot d\vec{A}$$

$$d\tau = r^2 \sin\theta dr d\theta d\phi$$

$$d\vec{A} = r^2 \sin\theta d\theta d\phi \hat{r}$$

$$\iint \frac{\hat{r}}{r^2} \cdot r^2 \sin\theta d\theta d\phi$$

=

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$$\oint \vec{E} \cdot d\vec{A} = \frac{\oint \rho dV}{\epsilon_0}$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2}$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \nabla \cdot \hat{r})$$

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2}$$

$$\rho = \vec{\nabla} \cdot \vec{E}$$

$$\iiint \vec{\nabla} \cdot \frac{\hat{r}}{r^2} dV = \oint \frac{\hat{r}}{r^2} \cdot d\vec{A}$$

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$d\vec{A} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

$$\oint \frac{\hat{r}}{r^2} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

=

$$\iiint_V \vec{\nabla} \cdot \frac{\hat{r}}{r^2} d^3r = \iint_{\partial V} \frac{\hat{r}}{r^2} \cdot d\vec{A}$$

$$d^3r = r^2 dr \sin\theta d\theta d\phi$$

$$d\vec{A} = r^2 \sin\theta d\theta d\phi \hat{r}$$

$$\iint \frac{\hat{r}}{r^2} \cdot r^2 \sin\theta d\theta d\phi \hat{r}$$

$$= 4\pi$$

2 konträrekte Resultate

$$\vec{\nabla} \cdot \frac{\hat{r}}{r^2} \quad \neq 0$$

$$\delta(x) : \int_{-\infty}^{\infty} \delta(x) dx = 1$$

~~$$\int_0^{\infty} \delta(x) dx = ?$$~~

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ x=0 & x=1 & x=2 & x=3 \end{array}$$

$$\rho = [\delta(x) + \delta(x-1) + \delta(x-2) + \delta(x-3)] \rho$$

$$\vec{E} = \sum \vec{E}_i$$

$$\vec{F}_{21} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

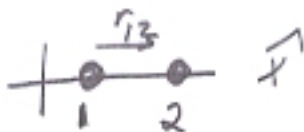
$\bullet^1 (x_1, y_1)$ $\bullet^2 (x_2, y_2)$

$\xrightarrow{\vec{r}_{12}}$ (\vec{r}_{ip})

$$\vec{r}_1 = x_1 \hat{x} + y_1 \hat{y}$$

$$\vec{r}_2 = x_2 \hat{x} + y_2 \hat{y}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1) \hat{x} + (y_2 - y_1) \hat{y}$$



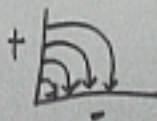
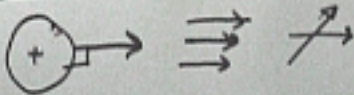
$$\vec{r}_2 = (2-1)\hat{x} = 1\hat{x}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{E} = \frac{1}{\rho} \hat{r}$$

$\vec{a} \rightarrow$ Acceleration

$$\vec{F} = \rho \vec{E}$$

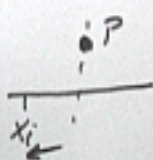


$$\vec{E}_p = \sum_{i=1}^n k \frac{q_i}{r_{ip}^2} \hat{r}_{ip}$$

$$\vec{E}_p = \int k \frac{dq_i}{r_{ip}^2} \hat{r}_{ip}$$

all q_i


$$\int_{L_i}^i \rightarrow \vec{r}$$



$$\vec{r}_i = x_i \hat{x} + 0 \hat{y}$$

$$\vec{r}_p = 0 \hat{x} + y_p \hat{y}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i \hat{x} + y_p \hat{y}$$



$$\vec{r}_i = x_i \hat{x} + 0 \hat{y}$$

$$\vec{r}_p = 0 \hat{x} + y_p \hat{y}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i \hat{x} + y_p \hat{y}$$

$$\vec{E}_p = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \lambda dx_i \frac{-x_i \hat{x} + y_p \hat{y}}{[x_i^2 + y_p^2]^{3/2}}$$

$$E = \int \frac{\lambda dx_i}{[x_i^2 + y_p^2]^{3/2}} (-x_i \hat{x} + y_p \hat{y})$$

$$\vec{E} = \int \lambda dx_i \frac{\vec{r}_{ip}}{|\vec{r}_{ip}|^3}$$



$|r_i|^{3/2}$



$$\vec{E} = \int_{-L/2}^{L/2} \frac{k \lambda dx_i}{[x_i^2 + y_p^2]^{3/2}} \hat{y}$$

$$= \cancel{k \lambda y_p} \hat{y} = k \lambda y_p \hat{y} \int_{-L/2}^{L/2} \frac{dx_i}{[x_i^2 + y_p^2]^{3/2}}$$

$$= k \lambda y_p \hat{y} \left[\frac{x_i}{y_p^2 \sqrt{x_i^2 + y_p^2}} \right]_{-L/2}^{L/2}$$

$$= k \lambda y_p \hat{y} \left[\frac{L}{y_p \left(\left(\frac{L}{2}\right)^2 + y_p^2 \right)^{3/2}} \right]$$

case: $y_p \gg \frac{L}{2}$ $y_p \rightarrow \infty$

$|r_i|^{3/2}$



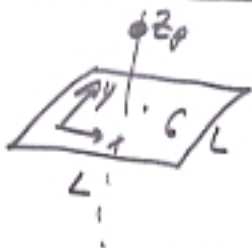
$$\vec{E} = \int_{-L/2}^{L/2} \frac{k \lambda dx_i}{[x_i^2 + y_p^2]^{3/2}} \hat{y}_p$$

$$= \cancel{k \lambda y_p} \hat{y}_p \int_{-L/2}^{L/2} \frac{dx_i}{[x_i^2 + y_p^2]^{3/2}}$$

$$= k \lambda y_p \hat{y}_p \left[\frac{x_i}{y_p^2 \sqrt{x_i^2 + y_p^2}} \right]_{-L/2}^{L/2}$$

$$= k \lambda y_p \hat{y}_p \left[\frac{L}{y_p \left(\left(\frac{L}{2}\right)^2 + y_p^2 \right)^{3/2}} \right]$$

Case: $y_p \gg \frac{L}{2}$ $y_p \rightarrow \infty$



$$Q = \iint \sigma \, dx \, dy$$

$$\sigma = \frac{Q}{L^2}$$

$$x_i, y_i, 0 \quad \vec{r}_i = x_i \hat{x} + y_i \hat{y} + 0 \hat{z}$$

$$\vec{r}_p = z_p \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E} = kC \int_{-L/2}^{+L/2} \int_{-L/2}^{+L/2} \frac{dx_i dy_i (-x_i \hat{x} - y_i \hat{y} + z_p \hat{z})}{[x_i^2 + y_i^2 + z_p^2]^{3/2}}$$



$$\Phi = \iint \sigma \, dx \, dy$$

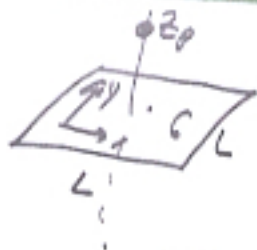
$$\sigma = \frac{\Phi}{L^2}$$

$$x_i, y_i > 0 \quad \vec{r}_i = x_i \hat{x} + y_i \hat{y} + z_p \hat{z}$$

$$\vec{r}_p = z_p \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E} = k\sigma \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{dx_i dy_i (-x_i \hat{x} + y_i \hat{y} + z_p \hat{z})}{[x_i^2 + y_i^2 + z_p^2]^{3/2}}$$



$$Q = \iint_S \sigma \, dxdy$$

$$\sigma = \frac{Q}{L^2}$$

$$x_i, y_i, 0 \quad \vec{r}_i = x_i \hat{x} + y_i \hat{y} + 0 \hat{z}$$

$$\vec{r}_p = z_p \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E} = k\sigma \int_{x=-L/2}^{L/2} \int_{y=-L/2}^{L/2} \frac{dxdy \cdot (-x_i \hat{x} + y_i \hat{y} + z_p \hat{z})}{[x_i^2 + y_i^2 + z_p^2]^{3/2}}$$

$$L = r_i p$$

$$G = \frac{Q}{4\pi a^2}$$



$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} + z \hat{z}$$

$$\vec{r}_p = 0 \hat{x} + 0 \hat{y} + z_p \hat{z}$$

$$\vec{r}_{ip} = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E} = k_c \int_0^a \int_0^{2\pi} \frac{-x_i \hat{x} - y_i \hat{y} + z_p \hat{z}}{r_i^3} \rho_p r_i dr_i d\theta$$

$$r_i = 0 \quad \theta = 0 \quad \left[\frac{x_i^2 + y_i^2 + z_p^2}{r_i^2} \right]^{3/2}$$

$$x_i = r_i \cos \theta_i$$

$$y_i = r_i \sin \theta_i$$

$$\vec{E} = k_c \int_0^a \frac{z_p \hat{z}}{[r_i^2 + z_p^2]^{3/2}} r_i dr_i$$

Next

$$r_i = 0$$

$$r = r_i p$$

$$G = \frac{Q}{4\pi a^2}$$



$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} + z \hat{z}$$

$$\vec{r}_p = 0 \hat{x} + 0 \hat{y} + z_p \hat{z}$$

$$\vec{r}_{ip} = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z}$$

$$\vec{E} = k_c \int_0^a \int_0^{2\pi} \int_0^{2\pi} \frac{-x_i \hat{x} - y_i \hat{y} + z_p \hat{z}}{r_i^2} r_i dr_i d\theta_i d\phi_i$$

$$r_i = 0 \quad \theta_i = 0 \quad \left[\frac{x_i^2 + y_i^2 + z_p^2}{r_i^2} \right]^{3/2}$$

$$x_i = r_i \cos \theta_i$$

$$y_i = r_i \sin \theta_i$$

$$\vec{E} = k_c \int_0^a \int_0^{2\pi} \int_0^{2\pi} \frac{z_p \hat{z}}{[r_i^2 + z_p^2]^{3/2}} r_i dr_i d\theta_i d\phi_i$$

NEXT

$$r_i = 0$$