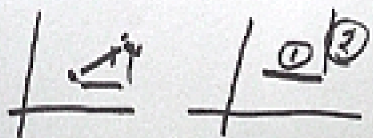
382
#2

$\oint \vec{V} \cdot d\vec{l} \qquad \vec{V} = ?\hat{\theta},$ [illegible]

$\int d(1\ \text{var})$

$\vec{V} = y^2\hat{x} + 2x(y+1)\hat{y}$

$(1,1) \rightarrow (2,2)$



$d\vec{l} = dx\,\hat{x}$

$$\int_{x=1,\,y=1}^{x=2} y^2\,dx = 1^2\int_1^2 dx = 1\cdot(2-1) = 1$$

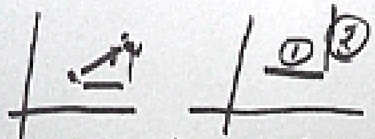
382
#2

$\oint \vec{v} \cdot d\vec{l}$ $\quad \vec{v} = ?\hat{\theta}, \vec{k}\times\vec{a}$

$\int d(1\ \text{var})$

$\vec{v} = y^2\hat{x} + 2x(y+1)\hat{y}$

$(1,1) \rightarrow (2,2)$



$d\vec{l} = dx\,\hat{x}$

$$\int_{x=1,\ y=1}^{x=2} y^2\,dx = 1^2\int_1^2 dx = 1\cdot(2-1) = 1$$

$$\vec r_2 = 2\hat x + 2\hat y$$

$$\vec r_{12} = \vec r_2 - \vec r_1 = 1\hat x + 1\hat y$$

$$\hat r_{12} = \frac{\vec r_{12}}{|\vec r_{12}|} = \frac{1}{\sqrt2}\hat x + \frac{1}{\sqrt2}\hat y$$

$$d\vec l = \frac{dx}{\sqrt2}\left(\hat x + \hat y\right) \qquad \int d(1\,yk)$$

$$\vec V = y^2\hat x + 2x(y+1)\hat y$$

$$\vec V = x^2\hat x + 2x(x+1)\hat y$$

$$\int_{11}^{22} \vec V\cdot d\vec l = \int_{x=1}^{x=2} \frac{dx}{\sqrt2}x^2 + \int_{x=1}^{x=2}\frac{dx}{\sqrt2}\cdot 2x(x+1)$$

$$= \frac{x^3}{3\sqrt2}\Big|_1^2 + \frac{2}{\sqrt2}\int (x^2+x)\,dx$$

$$= \left(\frac{8}{3\sqrt2} - \frac{1}{3\sqrt2}\right) + \frac{2}{\sqrt2}\left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_1^2$$

$$= \frac{7}{3\sqrt2} + \frac{2}{\sqrt2}\left(\frac83+\frac42-\left(\frac13+\frac12\right)\right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}}\left(\frac{14}{6} + \frac{12}{6} - \frac{2}{6} - \frac{3}{6}\right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}}\left(\frac{28}{6} - \frac{5}{6}\right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}}\left(\frac{23}{6}\right) = \frac{7}{3\sqrt{2}} + \frac{46}{6\sqrt{2}}$$

$$= \frac{7}{3\sqrt{2}} + \frac{23}{3\sqrt{2}} = \frac{30}{3\sqrt{2}} = \frac{10}{\sqrt{2}}$$

11

$V: -$

$$3\sqrt{2} + \frac{2}{\sqrt{2}}\left(\frac{8}{3} + \frac{4}{2} - \left(\frac{1}{3} + \frac{1}{2}\right)\right)$$

$$\vec{V}\cdot d\vec{A} = 4z\,dy\,dz$$

$$I = \int_{y=0}^{y=2}\int_{z=0}^{z=2} 4z\,dy\,dz$$

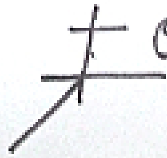
$$= 4(2)\int_{z=0}^{z=2} z\,dz$$

$$= 8\frac{z^2}{2}\Big|_0^2 = 8\cdot 2 = 16$$

$$\int_{y=0} \int_{z=0} 4z\,dy\,dz$$

$$= 4(2)\int_{z=0}^{z=2} z\,dz$$

$$= 8\frac{z^2}{2}\Big|_0^2 = 8\cdot 2 = 16$$



@ $y=2$ $\quad x=0,\ x=2$

$z=0,\ z=2$

$$d\vec{A} = dx\,dz\,\hat{y}$$

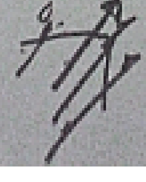
$$\vec{v}\cdot d\vec{a} = (x+2)\,dx\,dz$$

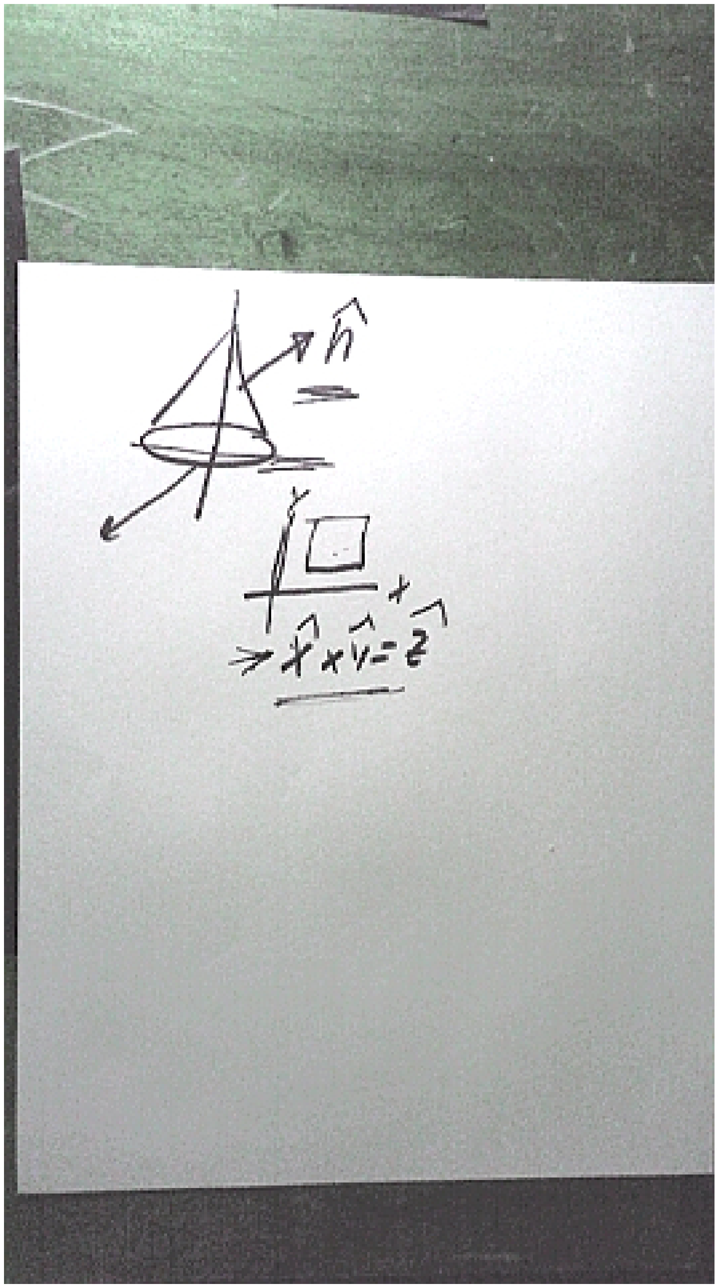
$$I = \int_{z=0}^{z=2}\int_{x=0}^{x=2} dx\,dz\,(x+2)$$

$$= 2\int_{x=0}^{x=2}(x+2)\,dx$$

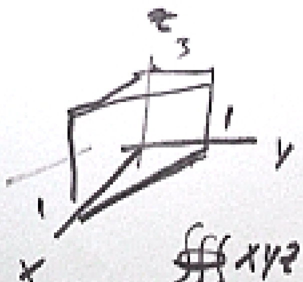
$$= 2\left(\frac{x^2}{2}+2x\right)\Big|_0^2 = 2(2+4)$$

$$= 2(6) = 12$$

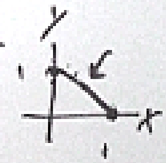



$\hat{n}$
y
x
$\hat{x} \times \hat{y} = \hat{z}$

$\iiint T \, d^3x \qquad \iiint \vec{T} \, d^3x$



xyz^2



$$\iiint xyz^2 (dx\,dy\,dz)$$

$$\boxed{= \int_{z=0}^{z=3} z^2 \, dz \int_{x=0}^{x=1} x \int_{y=0}^{y=1-x} y \, dx \, dy}$$

$$= \frac{z^3}{3}\Big|_0^3 \int x \, dx \left[\frac{y^2}{2}\right]_0^{1-x}$$

$$= 9 \int x\,dx \left[\frac{1-2x+x^2}{2} \right]$$

$$= \frac{1}{2} 9 \int x\,dx \,(1-2x+x^2)$$

$$= \frac{9}{2} \int_0^1 (x-2x^2+x^3)\,dx$$

$$\cancel{= \frac{9}{2}(1-2+1)} = 0$$

$$= \frac{9}{2} \left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4} \right)\Big|_0^1$$

$$= \frac{9}{2} \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right) \cdots \frac{3}{8}$$

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \iiint \rho(\vec{r})\, d^3 r$$

grad $\oint_a^b \vec{\nabla} T \cdot d\vec{L} = T(b) - T(a)$

$\vec{f} : \vec{\nabla} T = \vec{f}$

$\vec{f} = \vec{\nabla} T$

curls $\oiint (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = \oint \vec{V} \cdot d\vec{L}$

Parts: $\int \frac{d}{dx} (f \cdot g) \, dx = \int f \left(\frac{dg}{dx}\right) + \left(\frac{df}{dx}\right) g$

$\int_a^b f \frac{dg}{dx} dx = f g \Big|_a^b - \int_a^b \frac{df}{dx} g \, dx$