

382
#2

$$\oint \vec{v} \cdot d\vec{l} \quad \vec{v} = ? \hat{x} + \hat{y}$$

$$\int d(\text{ivar})$$

$$\vec{v} = y^2 \hat{x} + 2x(y+1) \hat{y}$$

$$(1,1) \rightarrow (2,2)$$

$$\int \hat{x} \quad \int \hat{y}$$

$$x = \int dx = dx \hat{x}$$
$$\int_{x=1, y=1}^2 y^2 dx = 1^2 \int dx = 1 \cdot (2-1) = 1$$

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#2

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$$\int d(\text{ivar})$$

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$$(1,1) \rightarrow (2,2)$$

$$\int_{(1,1)}^{(2,2)}$$

$$x=2 \quad \vec{dl} = dx \hat{x}$$
$$\int_{x=1, y=1}^2 y^2 dx = 1^2 \int_1^2 dx = 1 \cdot (2-1) = 1$$

$$\vec{r}_2 = 2\hat{x} + 2\hat{y}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = 1\hat{x} + 1\hat{y}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}$$

$$d\vec{l} = \frac{dx}{\sqrt{2}} (\hat{x} + \hat{y}) \int d(1+x)$$

$$\vec{V} = y^2 \hat{x} + 2x(y+1) \hat{y}$$

$$\vec{V} = x^2 \hat{x} + 2x(x+1) \hat{y} \quad \times 2x$$

$$\int \vec{V} \cdot d\vec{l} = \int_{x=1}^{x=2} \frac{dx}{\sqrt{2}} x^2 + \int_{x=1}^{x=2} \frac{dx}{\sqrt{2}} 2x(x+1)$$

$$= \frac{x^3}{3\sqrt{2}} \Big|_1^2 + \frac{2}{\sqrt{2}} \int (x^2 + x) dx$$

$$= \left(\frac{8}{3\sqrt{2}} - \frac{1}{3\sqrt{2}} \right) + \frac{2}{\sqrt{2}} \left(\frac{x^3}{3} + \frac{x^2}{2} \right) \Big|_1^2$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}} \left(\frac{8}{3} + \frac{4}{2} - \left(\frac{1}{3} + \frac{1}{2} \right) \right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}} \left(\frac{16}{6} + \frac{12}{6} - \frac{2}{6} - \frac{3}{6} \right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}} \left(\frac{28}{6} - \frac{5}{6} \right)$$

$$= \frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}} \left(\frac{23}{6} \right) = \frac{7}{3\sqrt{2}} + \frac{46}{\sqrt{2}}$$

$$= \frac{7}{3\sqrt{2}} + \frac{23}{3\sqrt{2}} = \frac{30}{3\sqrt{2}} = \frac{10}{\sqrt{2}}$$

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$$V: -\vec{\nabla} \times \vec{E}$$

$$\frac{7}{3\sqrt{2}} + \frac{2}{\sqrt{2}} \left(\frac{16}{3} + \frac{4}{2} - \left(\frac{1}{3} + \frac{1}{2} \right) \right)$$

$$\vec{v} \cdot d\vec{A} = 4z \, dy \, dz$$

$$I = \int_{y=0}^{y=2} \int_{z=0}^{z=2} 4z \, dy \, dz$$

$$= 4(2) \int_{z=0}^{z=2} z \, dz$$

$$= 8 \left. \frac{z^2}{2} \right|_0^2 = 8 \cdot 2 = 16$$

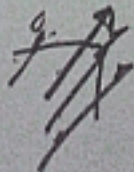
$$\begin{aligned}
 & \vec{v} = (x+2)\vec{i} \\
 & y=0 \quad z=0 \quad z=2 \\
 & = 4(2) \int_{z=0}^{z=2} z \, dz \\
 & = 8 \left. \frac{z^2}{2} \right|_0^2 = 8 \cdot 2 = 16
 \end{aligned}$$

$$\begin{aligned}
 & \text{+ } \odot y=2 \quad x=y+2 \\
 & \quad \quad \quad z=0, z=2
 \end{aligned}$$

$$d\vec{A} = dx dz \vec{j}$$

$$\vec{v} \cdot d\vec{A} = (x+2) dx dz$$

$$\begin{aligned}
 I &= \int_{z=0}^{z=2} \int_{x=0}^{x=2} dx dz (x+2) \\
 &= 2 \int_{x=0}^{x=2} (x+2) dx \\
 &= 2 \left(\frac{x^2}{2} + 2x \right) \Big|_0^2 = 2(2+4) \\
 &= 2(6) = 12
 \end{aligned}$$



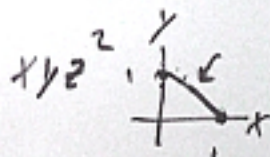
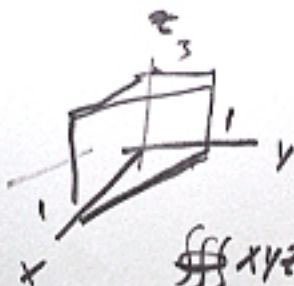


$\hat{x} \times \hat{y} = \hat{z}$

A hand-drawn diagram showing a square on a horizontal surface. A vertical line passes through the center of the square. Below the diagram, the equation $\hat{x} \times \hat{y} = \hat{z}$ is written and underlined.

$$\iiint_T d^3x$$

$$\iiint_T \vec{T} d^3x$$



$$\iiint_T xy z^2 (dx dy dz)$$

$$= \int_{z=0}^{z=3} z^2 dz \int_{x=0}^{x=1} x \int_{y=0}^{y=1-x} y dx dy$$

$$= \frac{z^3}{3} \Big|_0^3 \int x dx \left[\frac{y^2}{2} \right]_0^{1-x}$$

$$= \frac{9}{2} \int_0^1 x dx \left[\frac{1-2x+x^2}{2} \right]$$

$$= \frac{9}{2} \int_0^1 x dx (1-2x+x^2)$$

$$= \frac{9}{2} \int_0^1 (x-2x^2+x^3) dx$$

$$= \frac{9}{2} (1-2+1) = 0$$

$$= \frac{9}{2} \left(\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4} \right) \Big|_0^1$$

$$= \frac{9}{2} \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4} \right) \dots \frac{9}{2} \cdot 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \iiint \rho(\vec{r}) d^3r$$

$$\text{grad} \int_a^b \vec{\nabla} T \cdot d\vec{L} = T(b) - T(a)$$

$$\vec{f} : \vec{\nabla} T = \vec{f}$$

$$\vec{f} = \vec{\nabla} T$$

$$\text{curls} \oint (\vec{\nabla} \times \vec{V}) \cdot d\vec{A} = \oint \vec{V} \cdot d\vec{L}$$

$$\text{Parts: } \int_a^b \frac{d}{dx} (f \cdot g) = f \left(\frac{dg}{dx} \right) + \left(\frac{df}{dx} \right) g$$

$$\int_a^b f \frac{dg}{dx} dx = \int_a^b f g' - \int_a^b \frac{df}{dx} g dx$$