

$$r_p = x_p \hat{x} + y_p \hat{y} + z_p \hat{z}$$

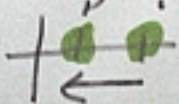
$$\hat{r}_p = \frac{\vec{r}_p}{|\vec{r}_p|}$$

$$|\vec{r}_p| = \sqrt{\vec{r}_p \cdot \vec{r}_p}$$

$$\oint d\vec{\ell} = dx \hat{x} + dy \hat{y} + dz \hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i \quad \vec{r}_i = 2\hat{x}$$

$$\vec{r}_p = 4\hat{x}$$



$$\vec{r}_{ip} = (1-2)\hat{x} = -\hat{x}$$

$$\vec{r}_i(2, 5, 7) \quad P(4, 6, 8)$$

$$\vec{r}_i = 2\hat{x} + 5\hat{y} + 7\hat{z}$$

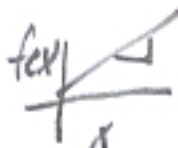
$$\vec{r}_p = 4\hat{x} + 6\hat{y} + 8\hat{z}$$

$$\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = (4-2)\hat{x} + (6-5)\hat{y} + (8-7)\hat{z}$$

$$= 2\hat{x} + 1\hat{y} + 1\hat{z}$$

$$\hat{r}_{ip} = \frac{\vec{r}_{ip}}{|\vec{r}_{ip}|} \quad |\vec{r}_{ip}| = \sqrt{2^2 + 1^2 + 1^2}$$
$$= \sqrt{4+1+1} = \sqrt{6} = 3$$

$$\hat{r}_{ip} = \frac{2}{3}\hat{x} + \frac{1}{3}\hat{y} + \frac{1}{3}\hat{z}$$

$f(x, y)$    $(y = mx + b)$   
 $\frac{dy}{dx} = m$

$$df = \left( \frac{df}{dx} \right) dx$$

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$$f(x, y, z)$$

$$\rightarrow df = \left( \frac{\partial f}{\partial x} \right) dx + \left( \frac{\partial f}{\partial y} \right) dy + \left( \frac{\partial f}{\partial z} \right) dz$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

$$df = \vec{\nabla} f \cdot d\vec{l}$$

$$f = r = \sqrt{x^2 + y^2 + z^2} \quad (x^2 + y^2 + z^2)^{1/2}$$

$$\vec{\nabla} f: \text{Diagram showing a vector pointing outwards from the origin in a 3D coordinate system.$$

$$\frac{\partial f}{\partial x} = \frac{1}{2} (x^2 + y^2 + z^2)^{-1/2} \cdot 2x$$

$$\vec{\nabla} f = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \quad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{\nabla} f = \frac{\vec{r}}{|\vec{r}|} = \hat{r} \quad \begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ |\vec{r}| &= r \end{aligned}$$

$$\vec{\nabla} r = \hat{r} \quad \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2 + y^2 + z^2}} = \hat{r}$$



$$S = \sqrt{x^2 + y^2}$$

$$\vec{\nabla} S = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + y^2}} \cdot 2x \hat{x}$$

$$+ \frac{y}{\sqrt{x^2 + y^2}} \hat{y} = \frac{x\hat{x} + y\hat{y}}{|S|}$$

$$= \frac{\vec{S}}{|S|} = \hat{S} \quad \text{for } \vec{S} = \vec{r}$$

$$f = e^{x/y^2} \quad \vec{\nabla} f = y^2 f_x \hat{x} + x^2 f_y \hat{y} + xy f_z \hat{z}$$
$$= f \left( \frac{f_x}{x} \hat{x} + \frac{f_y}{y} \hat{y} + \frac{f_z}{z} \hat{z} \right)$$

$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} + z_i \hat{z}$$

$$\vec{r}_p = x_p \hat{x} + y_p \hat{y} + z_p \hat{z} \quad i, p = ?$$

$$\vec{\nabla}_p r_{ip} = \frac{\partial}{\partial \vec{r}_p} r_{ip} = r_{ip} = (x_p - x_i)$$

$$r_{ip}^2 = (x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_i)^2$$

$$\begin{aligned} \vec{\nabla}_p r_{ip}^2 &= 2(x_p - x_i) \hat{x} \\ &\quad + 2(y_p - y_i) \hat{y} \\ &\quad + 2(z_p - z_i) \hat{z} \\ &= 2 \vec{r}_{ip} \end{aligned}$$

$$\vec{\nabla}_p \left( \frac{1}{r_{ip}} \right)$$

$$r_{ip} = \sqrt{(x_p - x_i)^2 + (y_p - y_i)^2 + (z_p - z_i)^2}$$

$$\frac{1}{r_{ip}} = [ ]^{-1/2}$$

$$\frac{1}{2} [ ]^{-3/2} (2(x_p - x_i)) \vec{x}$$

$$\dots$$
$$= 2 \frac{\vec{r}_{ip}}{|\vec{r}_{ip}|^3}$$

$$\vec{\nabla}_p (r_{ip}^n)$$

$$r_{ip}^n = [r_{ip}]^n$$

$$r_{ip}^n = (x_p - x_i)^n$$

$$\vec{\nabla}_p r_{ip}^n = n (x_p - x_i)^{n-1} \cdot \vec{1}$$

$$= n \cdot r_{ip}^{n-1} \vec{1}_{ip}$$