(251.02)

(1)

$\vec{r}_1 = -1\hat{x} + 2\hat{y}$

$\vec{r}_2 = 1\hat{x} + 2\hat{y}$

$\vec{r}_3 = 0\hat{x} + 0\hat{y}$

$\vec{r}_p = 5\hat{x} + 4\hat{y}$

$\vec{r}_{1p} = \vec{r}_p - \vec{r}_1 = (5-(-1))\hat{x} + (4-2)\hat{y}$
$= 6\hat{x} + 2\hat{y}$

$\vec{r}_{2p} = \vec{r}_p - \vec{r}_2 = (5-1)\hat{x} + (4-2)\hat{y}$
$= 4\hat{x} + 2\hat{y}$

$\vec{r}_{3p} = \vec{r}_p - \vec{r}_3 = (5-0)\hat{x} + (4-0)\hat{y}$
$= 5\hat{x} + 4\hat{y}$

$$\vec{r}_P = 5\hat{x} + 4\hat{y}$$

$$\vec{r}_{1P} = \vec{r}_P - \vec{r}_1 = (5 - {}^{-}1)\hat{x} + (4-2)\hat{y}$$
$$= 6\hat{x} + 2\hat{y}$$

$$\vec{r}_{2P} = \vec{r}_P - \vec{r}_2 = (5-1)\hat{x} + (4-2)\hat{y}$$
$$= 4\hat{x} + 2\hat{y}$$

$$\vec{r}_{3P} = \vec{r}_P - \vec{r}_3 = (5-0)\hat{x} + (4-0)\hat{y}$$
$$= 5\hat{x} + 4\hat{y}$$

$$\hat{r}_{1P} = \frac{\vec{r}_{1P}}{|\vec{r}_{1P}|} = \frac{6\hat{x}+2\hat{y}}{\sqrt{6^2+2^2}} = \frac{6\hat{x}+2\hat{y}}{6.3}$$
$$= .95\hat{x} + .32\hat{y}$$

$$\vec{E}_P = k\lambda\left[(+1)\frac{6\hat{x}+2\hat{y}}{[6^2+2^2]^{3/2}} + (-2)\frac{4\hat{x}+2\hat{y}}{[4^2+2^2]^{3/2}} + (+4)\frac{5\hat{x}+4\hat{y}}{[5^2+4^2]^{3/2}}\right]$$

$$kq_1\left[\frac{(-2)}{\sqrt{1^2+2^2}} + \frac{(-2)}{\sqrt{4^2+2^2}} + \frac{(+4)}{\sqrt{5^2+4^2}}\right]$$

$= 8990$

$W_1 = 0$

$W_2 = q V_{@2 \text{ from } 1}$

$r_{12} = \hat{r}_2 - \hat{r}_1 = (-1-1)\hat{x} + (2-2)\hat{y}$
$= -2\hat{x} + 0\hat{y}$

$|\hat{r}_{12}| = \sqrt{(-2)^2+0^2} = 2$

$W_2 = (-2)\left(\frac{+1}{2}\right) kq_1$
$= -8990$

$W_3 = q_3 V_{@3 \text{ from } 1} + q_3 V_{@3 \text{ from } 2}$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1 = (-1-1)\hat{x} + (2-2)\hat{y}$

$= -2\hat{x} + 0\hat{y}$

$|\vec{r}_{12}| = \sqrt{(-2)^2 + 0^2} = 2$

$$W_3 = (+4)_{\mu}\left(\frac{k\mu(1)}{\sqrt{1^2+2^2}}\right) + (+4)\frac{k\mu(-2)}{\sqrt{1^2+2^2}}$$

$$= k\mu\mu\left[\frac{4}{\sqrt{5}} + \frac{-8}{\sqrt{5}}\right] J$$

$899 \times 10^{-6}$

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$\vec{r}_1 = -1\hat{x} + 2\hat{y}$

$\vec{r}_2 = 1\hat{x} + 2\hat{y}$

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1 = (1-(-1))\hat{x} + 0\hat{y}$

$W_2 = q_2 V_{\text{at 2 from 1}}$

$$V_{\text{at 2 from 1}} = k\frac{q_1}{r_{21}} = k\mu \frac{1}{\sqrt{2^2+0^2}} = \frac{k\mu\, 1}{2}$$

$\vec{r}_{21} = 2\hat{x}$

$$= (k\mu\mu)\left[\frac{4}{\sqrt{5}} + \frac{-2}{\sqrt{5}}\right] \text{J}$$

$8990 \times 10^{-6}$

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$$W_2 = q_2 V_{\text{at } 2 \text{ from } 1}$$

$\vec{r}_1 = -1\hat{x} + 2\hat{y}$

$\vec{r}_2 = 1\hat{x} + 2\hat{y}$

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1 = (1-(-1))\hat{x} + 0\hat{y}$

$$V_{\text{at } 2 \text{ from } 1} = k\frac{q_1}{r_{21}} = k\mu \cdot \frac{1}{\sqrt{2^2 + 0^2}} = \frac{k\mu \cdot 1}{2}$$

$\vec{r}_{21} = 2\hat{x}$

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$$W_2 = q_2 V_{\text{at } 2 \text{ from } 1} = (-2\mu)\frac{k\mu \cdot 1}{2}$$

$$= (-2\mu)\,\frac{k\mu \cdot 1}{2}$$

$$= -k\mu\mu$$

$$= -8990 \times 10^{-6}\ \text{J}$$

(2) 220

$V = ax + b$


$\vec{E} = -\frac{\Delta V}{\Delta x}\hat{x}$

$= -a\hat{x}$

$V = ax + by + c$

$\vec{E} = -\frac{\Delta V}{\Delta x}\hat{x} - \frac{\Delta V}{\Delta y}\hat{y}$

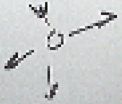
$\vec{E} = -a\hat{x} - b\hat{y}$

$V = ax^2 + by^3$ 22

$$\vec{E} = -\vec{\nabla} V$$

$$= -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y}$$

$$\vec{E} = -2ax\hat{x} - 3by^2\hat{y}$$



(3) 220

$\rho = \dfrac{Q}{\frac{4}{3}\pi a^3}$

$Q' = \rho \cdot$ Vol sphere

$= \dfrac{Q}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi a^3\right) = Q$

GS,

$r > a$

$\vec{E} \parallel \vec{A}$

$|\vec{E}|$ uniform

$\oint_E = \dfrac{Q_{enc}}{\varepsilon_0}$

out side $Q_{enc} = Q$

$\Phi_E = \sum_{\Delta A_i} \vec{E}_i \cdot \Delta\vec{A}_i = \sum_{\Delta A_i} E_i \Delta A_i$

$= E \sum \Delta A_i = E(4\pi r^2)$

$\vec{E} \perp \vec{A}$

$|\vec{E}|$ const

$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$

outside $Q_{enc} = Q$

$$\Phi_E = \sum_{\Delta A_i} \vec{E}_i \cdot \Delta\vec{A}_i = \sum_{\Delta A_i} E_i \Delta A_i$$

$$= E \sum \Delta A_i = E(4\pi r^2)$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

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Inside

$$Q_{enc} = \rho \cdot \tfrac{4}{3}\pi r^3$$
$$= Q \cdot \left(\frac{r^3}{a^3}\right)$$

On GS:
$\vec{E} \parallel \vec{A}$
$|\vec{E}|$ = uniform

$$\Phi_E = \sum_{\Delta A_i} \vec{E}_i \cdot \Delta\vec{A}_i$$
$$= E(4\pi r^2)$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}\left(\frac{r^3}{a^3}\right)$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3}\hat{r}$$

$$= \frac{4\pi\rho_0}{a}\int_0^a r^3\,dr$$

$$Q = \frac{4\pi\rho_0}{a}\cdot\frac{a^4}{4} \qquad \boxed{\rho_0 = \frac{Q}{\pi a^3}}$$

a) $r > a$


$$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = Q$$

On GS
$\vec{E} \parallel d\vec{A}$
E uniform

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$= \oint E\,dA$$

$$= E\oint dA = E(4\pi r^2)$$

$$\therefore E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi r^2\varepsilon_0}\,\hat{r}$$

$$\therefore E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\vec{E} = \frac{q}{4\pi r^2 \varepsilon_0}\hat{r}$$

on GS: $\vec{E} \parallel d\vec{A}$

$|\vec{E}|$ const

$$Q_{enc} = \iiint \rho\, d^3r$$

$$= 4\pi \int_0^r \rho_0 \frac{r}{a} \cdot r^2 dr$$

$$= \frac{4\pi\rho_0}{a} \int_0^r r^3 dr$$

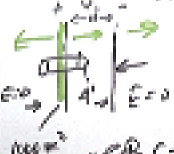
$$= \frac{4\pi\rho_0}{a} \cdot \frac{r^4}{4}$$

$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = E(4\pi r^2)$$

$$E(4\pi r^2) = \frac{4\pi\rho_0}{\varepsilon_0 a}\frac{r^4}{4}$$

$$\vec{E} = \frac{\rho_0 r^2}{4\varepsilon_0 a}\hat{r}$$

(4) $U = \frac{1}{2}CV^2$




$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

$Q_{enc} = \sigma A'$

$\sigma = \frac{Q}{Area}$

$\oint_S = EA'$

$EA' = \frac{\sigma}{\epsilon_0} A' \Rightarrow E = \frac{\sigma}{\epsilon_0}$

$\vec{E} = -\frac{dV}{dx}\hat{x}$

$|\Delta V| = \frac{\sigma}{\epsilon_0} d$

$\Delta V = Ed$

$C \equiv \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} d} = \epsilon_0 \frac{A}{d}$

$\hat{E} = -\frac{\partial V}{\partial x}\hat{x}$ $\quad EA = \frac{\sigma}{\epsilon_0}A \Rightarrow E = \frac{\sigma}{\epsilon_0}$

$|\Delta V| = \frac{\sigma}{\epsilon_0} d$

$\Delta V = Ed$

$C_{gap} = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} d} = \epsilon_0 \frac{A}{d}$

$U = \frac{1}{2} C V^2$ $\quad 8.8\times10^{-12}$

$= \frac{1}{2}\left(\epsilon_0 \left[\frac{1\times10^{3}}{1\times10^{-5}}\right]\right)(100)^2$

$= \frac{1}{2}\left(8.8\times10^{-12} \times 1\times10^{2}\right)1\times10^{4}$

$= \frac{1}{2}\, 8.8\times10^{-2} = 4.4\times10^{-2}\ J$

$C = 8.8\times10^{-12} \times \frac{1\times10^{3}}{1\times10^{-3}}$

$= 8.8\times10^{-6} f = \underline{8.8 \mu f}$

$K = 220$

$C = K C_{gap} = 1936 \mu f$

$C = 1 \times 8.85\times10^{-12} \times \frac{1\,m^2}{1\times10^{-3}}$

$= 8.85\times10^{-9}\,F = 8.85\,nF$

K = 200

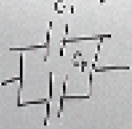
$= 1760\,nF$

Series

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C_{eq}} = \frac{2}{1760}$ ~~= [illegible] nf~~

$C_{eq} = 880\,nF$



$C_{eq} = C_1 + C_2$

$= 1760 \times 2 = 3520\,nF$

5 , 2

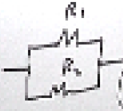
$\frac{1}{C_{eq}} = \frac{1}{5} + \frac{1}{2} = .2 + .5 = .7$

$C_{eq} = \frac{1}{.7} = 1.43\,\mu F$

(5) $R_1 = 100\,\Omega$ $R_2 = 300\,\Omega$


$R = R_1 + R_2$

$= 100 + 300 = 400\,\Omega$




$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{100} + \frac{1}{300}$$

$$= \frac{3}{300} + \frac{1}{300} = \frac{4}{300}$$

$$R_{eq} = \frac{300}{4} = 75\,\Omega$$

$$R_{ratio} = \frac{400\,\Omega}{75\,\Omega} = 5.3$$

(b)


$$C = \frac{Q}{V}$$

$$V = k\frac{Q}{a}$$

$$C = \frac{Q}{\frac{kQ}{a}} = \frac{a}{k}$$

$U = \frac{1}{2} C V^2$

On G.S:
$\vec{E} \parallel \vec{A}$ (CSA)

$\oint \vec{E} \cdot d\vec{A} = E(2\pi s h)$

$\sum_{\Delta A} \vec{E}_i \cdot \Delta\vec{A}_i = E(2\pi s h)$

$Q_{enc} = \lambda h$

$\vec{E} = \frac{\lambda h}{2\pi s h \epsilon_0} = \frac{\lambda}{2\pi \epsilon_0 s} \hat{s}$

$V = \int_a^b \frac{\lambda}{2\pi \epsilon_0 s} ds = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{b}{a}\right)$

[illegible]

$$q_{enc} = \lambda h$$

$$\vec{E} = \frac{\lambda h}{2\pi s h \epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$

$$V = \int_a^b \frac{\lambda}{2\pi\epsilon_0 s}\,ds = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

$$V = \frac{\lambda}{2\pi\epsilon_0}\sum\frac{\Delta s_i}{s_i} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{\lambda h}{\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0 h}{\ln\left(\frac{b}{a}\right)}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left[\frac{2\pi\epsilon_0 h}{\ln(b/a)}\right](V)^2$$