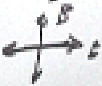
250

$$\begin{bmatrix} \vec{E} \\ \vec{B} \end{bmatrix} = \begin{bmatrix} E_m \\ B_m \end{bmatrix} \cos(kz - \omega t) \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

TEM



$$\frac{E}{B} = c$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\langle u_E \rangle = \frac{1}{2} \varepsilon_0 E^2 \underbrace{\langle \cos^2(\omega t) \rangle}_{\frac{1}{2}}$$

$$\langle u_M \rangle = \frac{B^2}{4\mu_0}$$

$$\langle u \rangle = \frac{1}{2} \varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

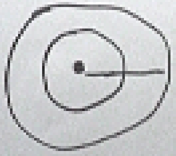
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\langle \vec{S} \rangle = \frac{E_m B_m}{2\mu_0} \hat{z}$$

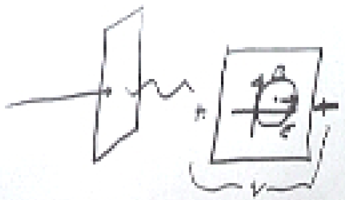
$I \quad \frac{\text{Power}}{\text{Area}}$

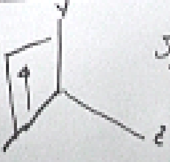
$$I = \langle \vec{S} \rangle \cdot \hat{n}$$

$$= c\langle u \rangle$$

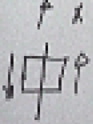


$$\frac{\langle \text{Power} \rangle}{4\pi R^2} = I$$





$$J_s = J_0 \cos(\omega t)$$

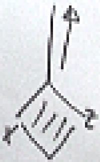


$$B = -\mu_0 \frac{J_s}{2}$$

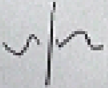
$$B = -\mu_0 \frac{J_0}{2} \cos(\omega t)$$

$$\vec{B}_r = -\frac{\mu_0}{2} J_0 \cos(kx - \omega t)\,\hat{y}$$

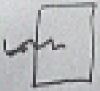
$$\vec{E}_r = \frac{\mu_0 c}{2} J_0 \cos(kx - \omega t)\,\hat{z}$$

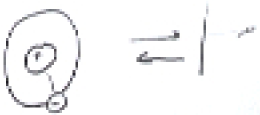


$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{\mu_0 c J_0^2}{4}\cos^2(kx-\omega t)$$



$$\langle \vec{S} \rangle \cdot \hat{n} = \frac{\mu_0 c J_0^2}{4}$$





$$F = k\frac{q_1 q_2}{r^2} = m\frac{v^2}{r}$$

$$\vec{L} = \vec{R} \times \vec{P}$$

$$F = \frac{L^2}{m_e r^3}$$

$$\frac{ke^2}{r^2} = \frac{L^2}{m_e r^3}$$

(17) Compute your result to the Balmer series and thus provide a value for the Rydberg constant R.

Early on, it was observed that a very good fit to the visible spectrum from hydrogen obeys the following relationship:

$$\frac{1}{\lambda} = R_\infty \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

A direct comparison with the Bohr model then gives the Rydberg constant:

$$R_\infty = \frac{E_\infty}{hc}$$

The Rydberg constant has the value: $10\,973\,731.6 \text{ m}^{-1}$

(18) Determine an expression for the fine structure constant $\alpha = v_1/c$ (which is equal to 1/137).

It is really remarkable that so many constants fit together to produce a simple result here. The speed of the electron in the first Bohr orbital can be easily obtained now:

$$L = m_e v r \Rightarrow v = \frac{L}{m_e r} \Rightarrow v_1 = \frac{nh}{2\pi m_e n^2 a_0} = \frac{h}{2\pi m_e a_0} \Rightarrow \frac{v_1}{c} = \frac{h}{2\pi m_e a_0 c} \equiv \alpha$$

It is not too hard to show $\alpha = \frac{e^2}{\left(\frac{h}{2\pi}\right)c}$ and both results provide a remarkable combination of constants into one easily remembered value (1/137).

$$\frac{1}{\lambda} = R_\infty \left[\frac{1}{n^2} - \frac{1}{m^2} \right]$$

$$R_\infty = \frac{E_\infty}{hc}$$

12:35 - 1:50