

(250) F15

$$\oint_{\mathcal{V}} \vec{E} \cdot d\vec{A} = \iiint \rho$$

$$\oint_{\mathcal{V}} \vec{E} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{enc}} = \iiint \rho d^3r$$

$$\left(\rho(r) = \frac{Q}{\frac{4}{3}\pi a^3}, \frac{4}{3}\pi r^3 \right) = 4\pi \int \rho(r) r^2 dr$$

$$d^3r = r^2 \sin\theta d\theta d\phi$$

$$Q_{\text{enc}} = \iiint \rho d^3r$$

$$= 4\pi \int \rho(r) r^2 dr$$

$$\textcircled{1a} \quad Q = \iiint \rho \, d^3r$$

$$= 4\pi \int \rho \, r^2 \, dr$$

$$\rho = C \cdot r$$

$$Q = 4\pi \int_0^a C \cdot r \cdot r^2 \, dr$$

$$= 4\pi C \int_0^a r^3 \, dr$$

$$= \pi C \left(\frac{a^4}{4} \right)$$

$$C = \frac{Q}{\pi \left(\frac{a^4}{4} \right)}$$

$$\rho = \frac{Q}{\pi \left(\frac{a^4}{4} \right)} \cdot r$$

$$\rho = c \cdot r \quad \text{⊙} \quad \Phi$$

$$Q = 4\pi \int_0^a c r \cdot r^2 dr$$

$$= 4\pi c \int r^3 dr \quad \int r^3 dr = \frac{r^4}{4}$$

$$= \pi c r^4 \Big|_0^a$$

$$= \pi c a^4 \quad c = \frac{Q}{\pi a^4}$$

$$\rho = \frac{Q}{\pi a^4} r$$

$$Q = \iiint \rho \cdot d^3r$$

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi r^3}$$



Choose GS of radius r
Centered on charge

$$\oint_{\text{GS}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{On GS: } \vec{E} \parallel \vec{A}$$

$|\vec{E}|$ uniform

$$\oint_{\text{GS}} \vec{E} \cdot d\vec{A}$$

$$= \oint_{\text{GS}} E dA$$

$$= E \oint_{\text{GS}} dA = E (4\pi r^2)$$

$$Q_{\text{enc}} = \iiint_V \rho \cdot d^3r \quad (\rho = Cr)$$

$$= 4\pi C \int_0^r r \cdot r^2 dr$$

$$E = \frac{\int \rho(r')}{\epsilon_0 (4\pi r^2)} = \frac{Cr}{4\epsilon_0} \hat{r}$$

outside: $\oint \mathbf{E} = E(4\pi r^2)$
 GS: $r > a$

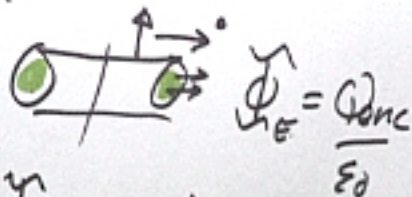
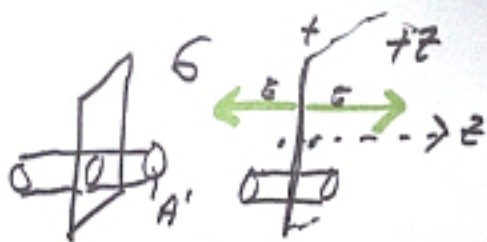


$$Q_{enc} = \iiint \rho d^3r$$

$$= 4\pi \int_0^a \rho r \cdot r^2 dr$$

$$= Q \quad ; \quad C = \frac{Q}{\pi(\epsilon_0^2)}$$

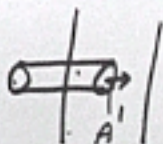
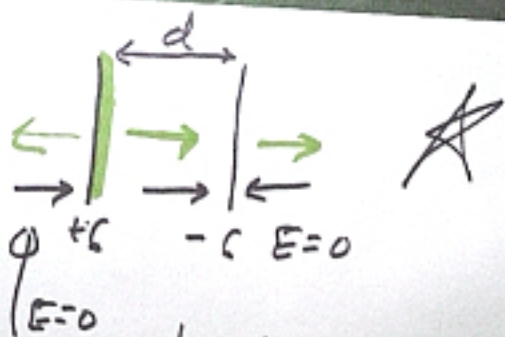




$$\Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

$$\Phi = 2EA' \quad Q_{enc} = \epsilon A'$$

$$2EA' = \frac{\epsilon A'}{\epsilon_0} \Rightarrow \vec{E} = \frac{\epsilon}{2\epsilon_0} \begin{cases} +\hat{z}: z > 0 \\ -\hat{z}: z < 0 \end{cases}$$



$$\oint \vec{E} \cdot d\vec{A} = EA' \quad Q_{\text{enc}} = \sigma A'$$

$$EA' = \frac{\sigma A'}{\epsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0} \hat{x} \text{ (correct)}$$

o otherwise