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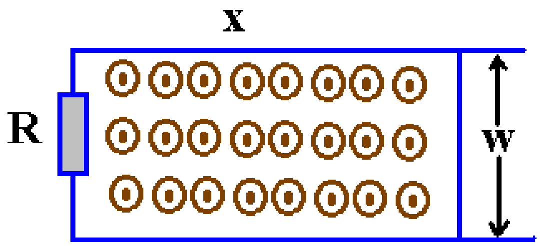
Faraday's Law of Induction

$$\mathcal{E} = -\frac{\Delta \Phi_M}{\Delta t} \quad \text{non calculus}$$

(1) Suppose you have the following situation: a conducting bar of length w is moving with a velocity v(t). The bar is lying on conducting rails. (a) find the current in the circuit when v(t)=v (v is constant) and (b) find the current in the circuit when the bar undergoes a constant acceleration a. **(c)** What about when $x = \frac{1}{2} a t^2$?



Solution: Suppose the bar is moving with a constant velocity v. Then

$$\frac{\Delta \Phi_M}{\Delta t} = B v w$$

so the induced emf has magnitude $\mathcal{E} = Bvw$. This produces a current in the circuit which is given by

$$I = \frac{Bvw}{R}$$

since by Ohm's law, $\mathcal{E} = IR$. Suppose however the bar underwent a constant acceleration a. Then

$$x = \frac{1}{2} a t^2$$

and the instantaneous velocity is at . This gives rise to an emf given by $\mathcal{E} = Batw$ and in this case, the current would be $I = \frac{Batw}{R}$. In this case, of course, the current would show a linear increase with time. Note that this problem works "the other way around" also.

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(2) Suppose a loop of wire of area A is rotating in a uniform magnetic field about an axis through the center of the loop and perpendicular to the magnetic field. Calculate the time dependence of the emf developed.

Solution: The magnetic flux is given by

$$\Phi_M = \vec{B} \cdot \vec{A} = BA\cos(\theta)$$

where θ is the angle between B and A. This angle, however, is so that $\theta=\omega t$ where $\omega=2\pi f$. Thus, the magnetic flux varies in time as

$$\Phi_M = BA\cos(\omega t)$$

and so

$$\frac{\Delta \Phi_M}{\Delta t} = -\omega BA\sin(\omega t) \quad .$$

According to Faraday's law, then the emf is given by

$$\mathcal{E} = \omega BA\sin(\omega t) \quad .$$

You can carry this a bit further by letting

$$\omega BA = \mathcal{E}_{max}$$

so you can say

$$\mathcal{E} = \mathcal{E}_{max}\sin(\omega t) \quad .$$

Now show 2 other ways to produce an emf.

We need to introduce a very important concept now, that of inductance (L). Inductance is defined through Faraday's law and there are two types of inductance: namely self inductance (L) or mutual inductance (M). We'll talk about M later. Suppose we have a coil of N turns and each turn has a magnetic flux $\Phi_{M,1}$. According to Faraday's law:

$$\mathcal{E} = -N\frac{\Delta \Phi_{M,1}}{\Delta t}$$

We want to make a proportionality between the flux in the coil and where it comes from (namely, the fact that the coils are carrying a current). We'll call the constant of proportionality the inductance of the circuit. Thus we can write

$$\mathcal{E} = -N\frac{\Delta \Phi_{M,1}}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

We see then that we could write in this special case the inductance as

$$L = \frac{N\Phi_{M,1}}{I}$$

or, in a more general sense, we could write something about a "self-induced" emf:

$$L = \frac{-\mathcal{E}}{\left[\frac{\Delta I}{\Delta t}\right]} \quad .$$

which shows more clearly the units of L are V/(A/s) = 1 Henry (H).

One way to measure inductance is this: connect a voltmeter across a coil, then change the current input into the coil from 0A to 1A in a time of 1s. If you have an inductance of 1 H, then you should measure a potential difference across the coil of 1 V.

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(3) Calculate the inductance of a uniformly wound solenoid with N turns and length w and cross sectional area A. Assume w is long compared to the radius and that the core of the solenoid is in air.

Solution: We have found the magnetic field inside the solenoid is

$$B=\mu_0 n I=\mu_0\left[\frac{N}{w}\right]I \quad .$$

We can now find the magnetic flux through ***each*** turn to be

$$\Phi_M=BA=\mu_0\left[\frac{NA}{w}\right]I \quad .$$

We can now find the inductance from the first definition of inductance:

$$L=\frac{N\Phi_M}{I}=\frac{\mu_0 N^2 A}{w}=\frac{\mu_0 (nw)^2 A}{w}=\mu_0 n^2[Aw]=\mu_0 n^2[\text{volume}]$$

Suppose the solenoid had $A=4\text{x}10^{-4}$ m^2, N=300 and w=0.25m. Then $L=1.81$ $\text{x}10^{-4}$ H. Now if you decrease I at the rate of 50000 A/s, this will induce an emf of 9.05V.

Be very careful how you turn off your magnets! It is easy to destroy controlling circuitry!

(4) Magnetic energy. Assume that we connect a current source to an inductor of self-inductance L and increase the current from zero at a constant rate $\frac{\Delta I}{\Delta t}$. At any point in time there will be an emf generated due the Faraday's law. In a sense, then the product $\mathcal{E} I[\Delta t]$ will be equal to energy, which is, in fact, a magnetic energy stored in the inductor when it carries a current I. On the face of it, it would seem that this energy would be given by

$$U_M = L\left[\frac{\Delta I}{\Delta t}\right][I\Delta t] = LI[\Delta I] \quad .$$

The result is actually that to establish a current I in the solenoid, the amount of stored energy is equal to $U_M = \frac{1}{2} L I^2$. This magnetic energy is as important for understanding magnetism as the electrostatic energy U_E was to understand electrostatics.

Suppose the solenoid in problem (3) has a current I through it. Calculate the magnetic energy and the ***magnetic energy density***.

Solution: we had for the solenoid that

$$L = \mu_0 n^2 [Aw]$$

so when the solenoid carries a current I, it holds an amount of magnetic energy equal to

$$U_M = \frac{1}{2}\mu_0 n^2 I^2 [Aw] \quad .$$

But $B = \mu_0 n I$ so we find that the magnetic energy is given by

$$U_M = \frac{1}{2}\left[\frac{B^2}{\mu_0}\right][Aw] \quad .$$

Thus the magnetic energy density is given by

$$u_M = \frac{U_M}{Aw} = \frac{B^2}{2\mu_0} \quad .$$

This is as important of a result as was the electrostatic energy density $u_E = \frac{1}{2}\epsilon_0 E^2$ and it has SI units of $\frac{J}{m^3}$.

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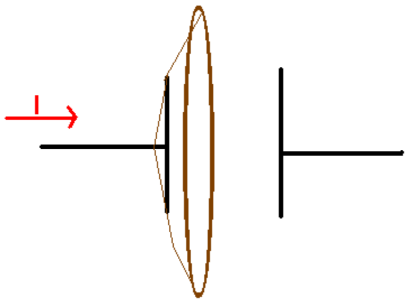
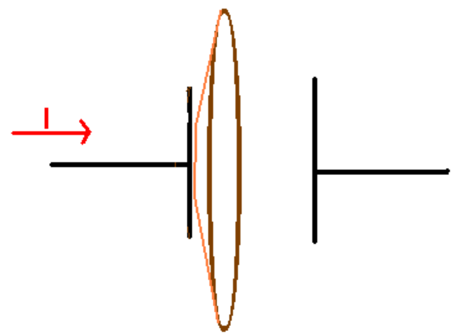
(5) **Displacement current: Maxwell's correction to Ampere's law**. Consider the capacitor shown below. The capacitor is charging with a current I and if we apply Ampere's law, then we can calculate the magnetic field easily enough to be



$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

Now suppose we changed the loop to look like that shown to the right. Here, According to Ampere's law then $I_c=0$ so B must be zero. With the same situation, we see that two different values for the magnetic field result.



Let's see how Maxwell fixed this problem.

We know that a changing magnetic flux produces an emf which is closely related to an electric field (which, granted, is different from other electric fields since it does not arise from charges, among other things). It is a reasonable guess that a changing electric flux would produce a magnetic field. The correction term needed is of the form

$$I_{displacement} = \epsilon_0 \left[\Delta \Phi_E / \Delta t \right] \quad \text{non-calculus}$$

The needed correction to Ampere's law is then

$$\sum_{curve} \vec{B}_i \cdot \Delta \vec{s}_i = \mu_0 \left(I_c + I_d \right) \quad \text{(non-calculus version)}$$

The d stands for the "displacement current" which is only very important when dealing with ac circuits. This is why we could use Ampere's law for magnetostatic problems: $I_d=0$ for static situations.

I have made an animation which shows the deformation of this Amperian Loop.