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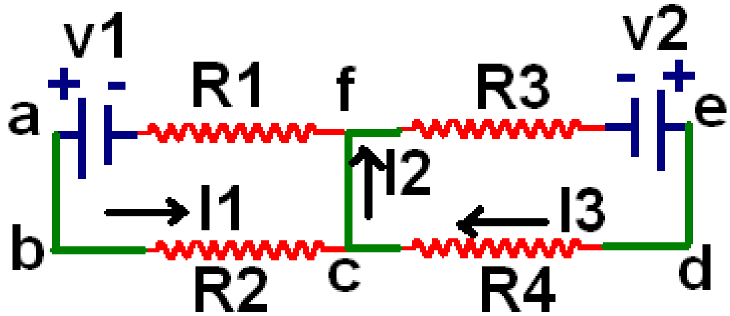
Consider the circuit shown below. Find the currents in each branch.



We have then 2 loops and 1 junction. Kirchoff's laws say:

$$(abcfa): v_1 - I_1 R_1 - I_1 R_2 = 0$$

$$(edcfe): v_2 - I_3 R_4 - I_3 R_3 = 0$$

$$@c: I_1 + I_3 - I_2 = 0$$

v1-I1*R1-I1*R2=0
v2-I3*R4-I3*R3=0
I1+I3-I2=0
Solutions for currents:

$$I_1 = \frac{V_1}{R_1 + R_2} : I_2 = \frac{V_1}{R_1 + R_2} + \frac{V_2}{R_3 + R_4} : I_3 = \frac{V_2}{R_3 + R_4}$$

The answer should not be too much of a surprise.

Note that at Wims, you would enter:
V1 - I1 * R1 - I1 * R2 = 0
V2 - I3 * R4 - I3 * R3 = 0
I1 + I3 - I2 = 0
parameters: R1,R2, R3, R4, V1, V2
Solution:

{ $i_1 = v_1/(r_2+r_1)$, $i_2 = (r_2\ v_2+r_1\ v_2+(r_4+r_3)\ v_1)/(r_2(r_4+r_3)+r_1(r_4+r_3))$, $i_3 = v_2/(r_4+r_3)$ }

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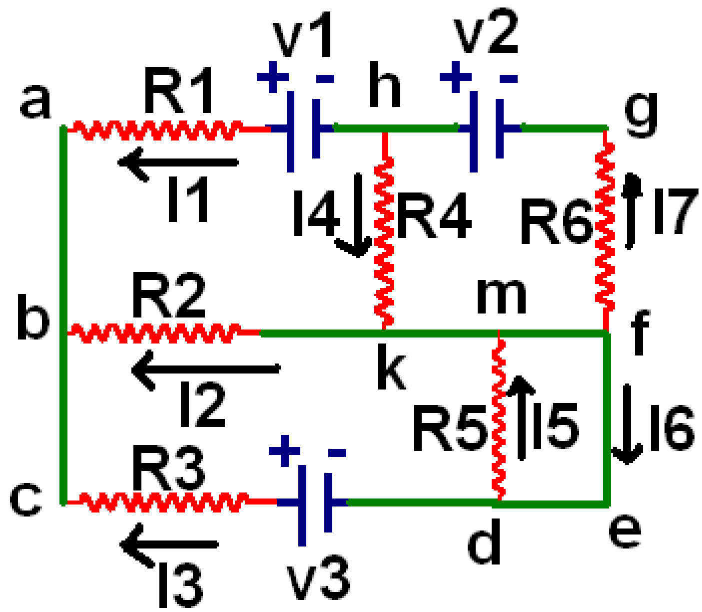


Needed: 4 loops, 3 junctions

(abkha): I2*R2+I4*R4+V1-I1*R1=0

(bcdmb): I3*R3-V3-I5*R5-I2*R2=0

(hkfgh): -I4*R4-I7*R6+V2=0

(defmd): I5*R5=0

@h: I7-I1-I4=0

@b: I1+I2+I3=0

@kmf: I5-I2-I6+I4-I7=0

At Wims, enter:

I2*R2+I4*R4+V1-I1*R1=0

I3*R3-V3-I5*R5-I2*R2=0

I4*R4-I7*R6+V2=0

I5*R5=0

I7-I1-I4=0

I1+I2+I3=0

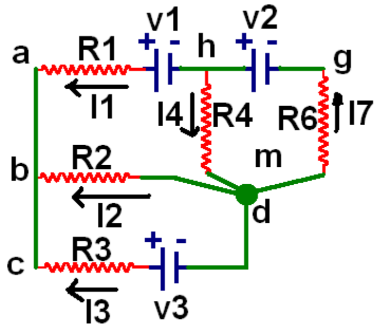
I5-I2-I6+I4-I7=0

With parameters: R1,R2, R3, R4, R5, R6, V1, V2, V3

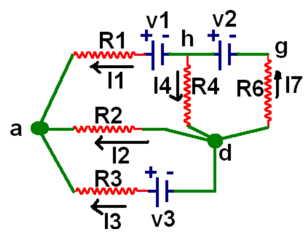
The result is a bit nasty:

{ i_1 = $(-r_2\, r_6\, v_3+r_4(r_2(v_2-v_3)+r_3\, v_2)+(r_3\, r_6+r_2\, r_6+(r_3+r_2)\, r_4)\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$, $i_2 = -(r_4(r_6\, v_3+r_1\, v_3+r_3\, v_2)+r_1\, r_6\, v_3+(r_3\, r_6+r_3\, r_4)\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$, $i_3 = -(r_4(-r_6\, v_3-r_1\, v_3+r_2(v_2-v_3))-r_2\, r_6\, v_3-r_1\, r_6\, v_3+(r_2\, r_6+r_2\, r_4)\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$, $i_4 = -(r_2(-r_6\, v_3-r_3\, v_2-r_1\, v_2)-r_1\, r_3\, v_2+(r_3\, r_6+r_2\, r_6)\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$, $i_5 = 0$, $i_6 = -(r_4(-r_6\, v_3-r_1\, v_3+r_2(v_2-v_3))-r_2\, r_6\, v_3-r_1\, r_6\, v_3+(r_2\, r_6+r_2\, r_4)\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$, $i_7 = (r_4(r_2(v_2-v_3)+r_3\, v_2)+r_2(r_3\, v_2+r_1\, v_2)+r_1\, r_3\, v_2+(r_3+r_2)\, r_4\, v_1)\, /(r_4(r_2(r_6+r_3+r_1)+r_3\, r_6+r_1\, r_3)+r_2(r_3\, r_6+r_1\, r_6)+r_1\, r_3\, r_6)$ }

You can note that the single solution here for I5 is zero. This is because the wire (defm) shorts out R5. The circuit is therefore can be simplified. In a simpler form, it looks like this:



We can also simplify the section (abc) so that the circuit looks like this:



Here, we need 3 loops and 2 junctions:
(ada): +I3*R3-V3-I2*R2=0
(adha): I2*R2+I4*R4+V1-I1*R1=0
(hdgh): -I4*R4-I7*R6+V2=0
@a: I1+I2+I3=0
@d: I4-I2-I7-I3=0

At Wims:
I3*R3 - V3 - I2*R2 = 0
I2*R2 + I4*R4 + V1 - I1*R1 = 0
-I4*R4-I7*R6+V2=0
I1+I2+I3=0
I4-I2-I7-I3=0
Parameters: R1,R2, R3, R4,R6, V1, V2, V3
If you solve these, the results are fairly nasty.

$$\{\ i_1 = -(r_2(r_6+r_4)\ v_3+r_3((-r_6-r_4)\ v_1-r_4\ v_2)+r_2((-r_6-r_4)\ v_1-r_4\ v_2))\ /(r_3(r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)+r_2(r_4(r_6+r_1)+r_1\ r_6)),\ i_2 = -((r_4(r_6+r_1)+r_1\ r_6)\ v_3+r_3(r_4\ v_2+(r_6+r_4)\ v_1))\ /(r_3(r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)+r_2(r_4(r_6+r_1)+r_1\ r_6)),\ i_3 = ((r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)\ v_3+r_2((-r_6-r_4)\ v_1-r_4\ v_2))\ /(r_3(r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)+r_2(r_4(r_6+r_1)+r_1\ r_6)),\ i_4 = (r_2\ r_6\ v_3+r_3(r_2\ v_2+r_1\ v_2-r_6\ v_1)+r_2(r_1\ v_2-r_6\ v_1))\ /(r_3(r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)+r_2(r_4(r_6+r_1)+r_1\ r_6)),\ i_7 = -(r_2\ r_4\ v_3+r_3(-r_4\ v_2-r_2\ v_2-r_1\ v_2-r_4\ v_1)+r_2(-r_4\ v_2-r_1\ v_2-r_4\ v_1))\ /(r_3(r_2(r_6+r_4)+r_4(r_6+r_1)+r_1\ r_6)+r_2(r_4(r_6+r_1)+r_1\ r_6))\ \}$$

What if, however, you now asked a question such as: supposing all resistors are the same, what value of V1 would cause I1 to become zero?

At Wims, enter:
I3*R - V3 - I2*R = 0
I2*R + I4*R + V1 - I1*R = 0
-I4*R-I7*R+V2=0
I1+I2+I3=0
I4-I2-I7-I3=0
Parameters: R, V1,V2,V3
I'll assign a value of R to each of these resistors. The result is this:
$\{ i_1 = -(v_3-v_2-2v_1)/(4r),\ i_2 = -(3v_3+v_2+2v_1)/(8r),\ i_3 = (5v_3-v_2-2v_1)/(8r),\ i_4 = (v_3+3v_2-2v_1)/(8r),\ i_7 = -(v_3-5v_2-2v_1)/(8r) \}$

Notice that $I_1 = \frac{-\left(V_3 - V_2 - 2V_1\right)}{4} R$

If we want I_1 to be zero, then we solve:

$$V_3 - V_2 - 2V_1 = 0 \Rightarrow V_1 = \frac{V_3 - V_2}{2}$$

Notice for this particular case, we would have I3=-I2 as is required. Also note that I4 and I7 are also the same. Essentially we have broken the circuit into two branches. You can now find the power dissipated in each element of the circuit and the total power dissipated.

You can also find this out at Wims: Enter:
I3*R - V3 - I2*R = 0
I2*R + I4*R + V1 - I1*R = 0
-I4*R-I7*R+V2=0
I1+I2+I3=0
I4-I2-I7-I3=0
I1=0
parameters: R, V2,V3

Solution: $\{ i_1 = 0,\ i_2 = -v_3/(2r),\ i_3 = v_3/(2r),\ i_4 = v_2/(2r),\ i_7 = v_2/(2r),\ v_1 = (v_3-v_2)/2 \}$

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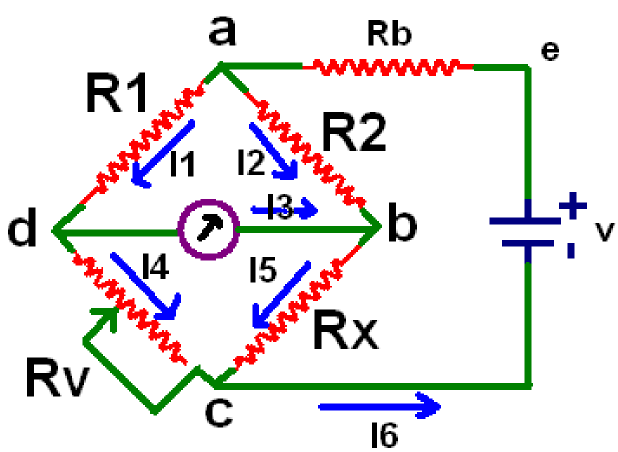
The Wheatstone Bridge Circuit:

Measurement of the unknown resistance R_x.



When the bridge is balanced, I3 is zero which means there is no potential difference between d and b.

(adcea): -I1*R1-I4*Rv+V-I6*Rb=0

(abcea): -I2*R2-I5*Rx+V-I6*Rb=0

(adba): -I1*R1-I3*R+I2*R2=0

@a: I6-I1-I2=0

@c: I4+I5-I6=0

If at Wims you enter:

-I1*R1-I4*S+V-I6*B=0

-I2*R2-I5*X+V-I6*B=0

-I1*R1-I3*R+I2*R2=0

I6-I1-I2=0

I4+I5-I6=0

Parameters:R,R1, R2, V, B, S,X

You will find that the system has infinite solutions. However, if the bridge is balanced, then I3=0. So that can be added to the system of equations, but note that if I3=0 then I1=I4 and I2=I5. Add these additional two conditions to the solver gives for I3:

$$i_3 = v(r_2\ s-r_1\ x)/(s(r\ x+r\ r_2+b\ r)+r_1(r\ x+r\ r_2+b\ r)+b(r\ x+r\ r_2))$$

Which means now that since I3 is zero, we must have:

The unknown resistance: $R_2 R_v - R_1 R_x = 0 \Rightarrow R_x = \dfrac{R_2}{R_1} R_v$.

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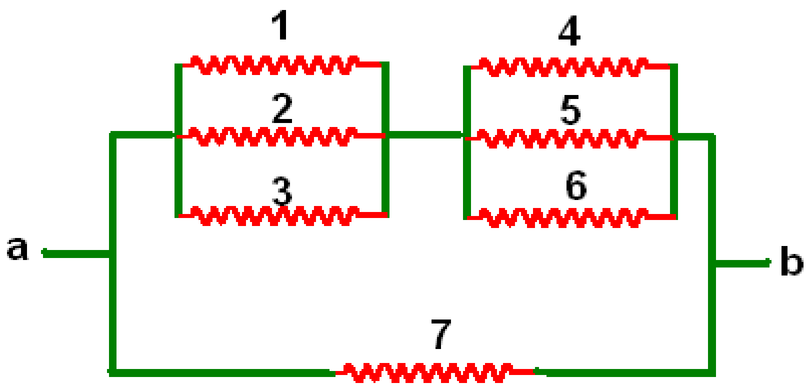
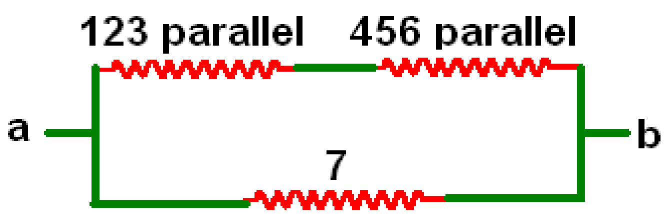
Reduction of circuits

It is possible with many simple circuits to reduce the circuits without Kirchoff's laws. Let me show you some examples of how to do this:

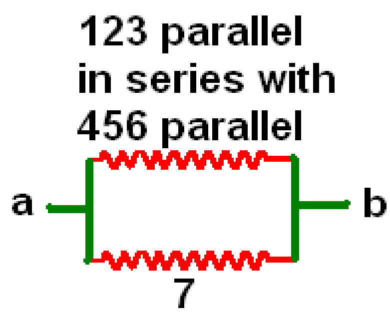


Reduce this circuit in a step-wise method recognizing series and parallel combinations of resistors:

Step 1:



Step2:



The last step in finding the equivalent resistance is then clear. If you would like to see a numerical example of this, let the resistors be the value of the number that I've assigned to them. Then

$$123\,\text{parallel} = \frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}} = \frac{1}{\frac{6}{6}+\frac{3}{6}+\frac{2}{6}} = \frac{1}{\frac{11}{6}} = \frac{6}{11}$$

$$456\,\text{parallel} = \frac{1}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}} = \frac{1}{\frac{3}{120}+\frac{24}{120}+\frac{20}{120}} = \frac{1}{\frac{74}{120}} = \frac{120}{74}$$

$$(123\,\text{parallel})\,\text{series}\,(456\,\text{parallel}) = \frac{6}{11} + \frac{120}{74} = \frac{444+1320}{814} = \frac{1764}{814}$$

The final step is to calculate the equivalent resistance of the last 2:

$$R_{eq} = \frac{1}{\frac{1764}{814} + \frac{1}{7}} = \frac{1}{\frac{12348+814}{5698}} = \frac{5698}{13162} = 0.43\,\Omega$$

You can work backwards from here to find the current through each resistor. It is somewhat important to be able to recognize these series and parallel combinations of resistors.

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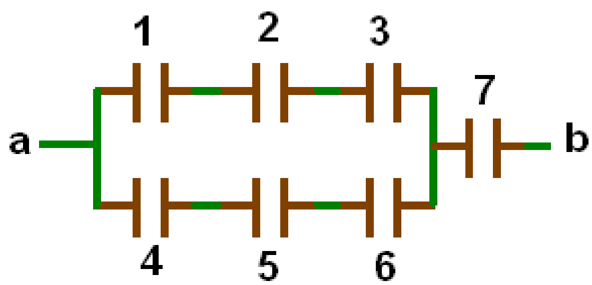
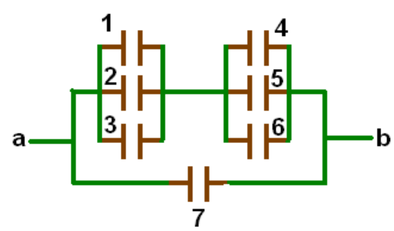
The same technique also works for capacitors.
in fact, it the same only different …. what is the equivalent capacitance of this circuit?

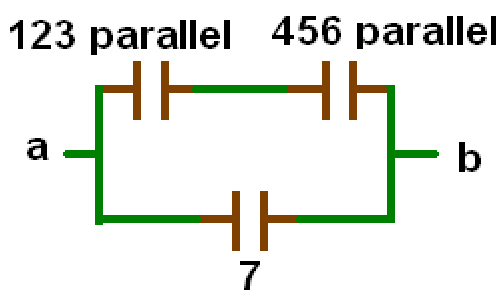


where each of the capacitances is given in μf? (without doing further calculations)
The answer after you think about it for a while is clearly 0.43 μf.

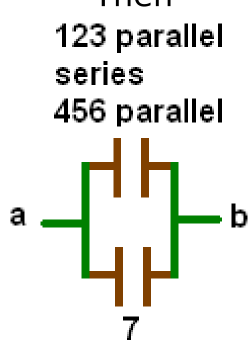
Let's do the same thing for a similar arrangement of capacitances:



The first reduction is:



Then



You can calculate this equivalent capacitance:

$$(123\,\text{parallel}) = 1+2+3 = 6\,\mu\text{f}$$

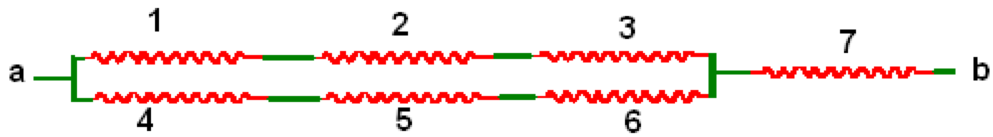
$$(456\,\text{parallel}) = 4+5+6 = 15\,\mu\text{f}$$

$$(123\,\text{parallel})\,\text{series}\,(456\,\text{parallel}) = \frac{1}{\frac{1}{6}+\frac{1}{15}} = \frac{1}{\frac{15}{90}+\frac{6}{90}} = \frac{90}{21}$$

The equivalent capacitance is then given by:

$$\frac{90}{21} + 7 = 11.29\,\mu f$$

Ok, what about this circuit



Without calculation, if the numbers represent Ohms, then the equivalent resistance is

11.29Ω

Unfortunately, these types of identifications (i.e. the connection between capacitive and resistive circuits in the analysis) is of limited utility which is why they're probably not often shown.