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Supplementary problems for electrostatics

1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m . One vertex is at (-1/2 ,0) and the second vertex is located at (+1/2,0) with the third vertex along the +y axis at (0,y). Find the electric field 1/2 way up the height of the triangle.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

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1. Three identical charges each with $q=1\mu C$ are arranged at the vertices of an equilateral triangle with a side of length 1m . One vertex is at (-1/2 ,0) and the second vertex is located at (+1/2,0) with the third vertex along the +y axis at (0,y). Find the electric field 1/2 way up the height of the triangle.

The hard part here is to locate coordinates of the point. Note that this is not the center of the triangle. Let the triangle have a side of length L. Locate one point at the origin. Locate the second point at (L,0). We need to find the coordinates of the center, and the 3rd point. The x coordinate of the 3rd point is x=L/2. The y coordinate is then given by:

$$L^2=\left(\frac{L}{2}\right)^2+y^2\Rightarrow y^2=L^2-\frac{L^2}{4}=3\frac{L^2}{4}\Rightarrow y=\frac{\sqrt{3}}{2}L\ .$$

The coordinate of the point is:

$$p=\left(\frac{L}{2},\frac{\sqrt{3}}{4}L\right)\ .$$

In our present case, L=1 m. We thus have the coordinates for this particular orientation:

$$1:(1\mu;-0.5,0),2:(1\mu;0.5,0),3:(1;0,\frac{\sqrt{3}}{2}),p:(0,\frac{\sqrt{3}}{4})$$

Calculate the following vectors: $\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_p,\vec{r}_{1p},\vec{r}_{2p},\vec{r}_{3p}$. These vectors are:

$$\vec{r}_p=0\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_1=-.5\hat{x}+0\hat{y}:\vec{r}_{1p}=\vec{r}_p-\vec{r}_1=0\hat{x}+\frac{\sqrt{3}}{4}\hat{y}-(-.5\hat{x}+0\hat{y})=0.5\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_2=1\hat{x}+0\hat{y}:\vec{r}_{2p}=\vec{r}_p-\vec{r}_i=\left(0\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-(.5\hat{x}+0\hat{y})=-0.5\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_3=0\hat{x}+\frac{\sqrt{3}}{2}\hat{y}:\vec{r}_{3p}=\vec{r}_p-\vec{r}_i=\left(0\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-\left(0\hat{x}+\frac{\sqrt{3}}{2}\hat{y}\right)=0\hat{x}-\frac{\sqrt{3}}{4}\hat{y}$$

Now write E directly.

$$\vec{E}=k\mu\left[\frac{.5\hat{x}+\frac{\sqrt{3}}{4}\hat{y}}{\left[\frac{1}{4}+\frac{3}{16}\right]^{3/2}}+\frac{-.5\hat{x}+\frac{\sqrt{3}}{4}\hat{y}}{\left[\frac{1}{4}+\frac{3}{16}\right]^{3/2}}+\frac{0\hat{x}-\frac{\sqrt{3}}{4}\hat{y}}{\left[\frac{3}{16}\right]^{3/2}}\right]=0\hat{x}+\frac{\sqrt{3}}{4}k\mu\hat{y}\left[\frac{2}{\left[\frac{7}{16}\right]^{3/2}}-\frac{1}{\left[\frac{3}{16}\right]^{3/2}}\right]$$

$$\vec{E}=0\hat{x}+k\mu\hat{y}(1.496-5.333]=0\hat{x}-3.84k\mu=0\hat{x}-34521\hat{y}\frac{N}{C}$$

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2. A square has charges at the following locations and values: (1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

Notice that if $\vec{r}_p = 0\hat{x} + 0\hat{y}$, then $\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = \vec{0} - \vec{r}_i = -\vec{r}_i$. We can use this to write down the electric field directly:

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+1\hat{y})}{[1^2+1^2]^{3/2}} + \frac{2(-1\hat{x}+1\hat{y})}{[2]^{3/2}} + \frac{3(-1\hat{x}-1\hat{y})}{2\sqrt{2}} + \frac{4(\hat{x}-\hat{y})}{2\sqrt{2}}\right]$$

$$\Rightarrow \vec{E}_p = \frac{k\mu}{2\sqrt{2}}[\hat{x}(1-2-3+4)+\hat{y}(1+2-3-4)] = \frac{k\mu}{2\sqrt{2}}(-4\hat{y}) = \frac{-2k\mu}{\sqrt{2}}\hat{y} = -\sqrt{2}k\mu\hat{y} = -12713\hat{y}\frac{N}{C}$$

3. Three charges are arranged as follows:(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

This satisfies the condition $\vec{r}_p = \vec{0}$. We can then write the field directly.

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+0\hat{y})}{[1^2+0^2]^{3/2}} + \frac{-1(-1\hat{x}+0\hat{y})}{1} + 1(0\hat{x}-1\hat{y})\right] = k\mu[\hat{x}(1+1+0)+\hat{y}(0+0-1)] = k\mu[2\hat{x}-1\hat{y}]$$

$$\Rightarrow \vec{E}_p = [17980\hat{x} - 8990\hat{y}]\frac{N}{C}$$

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.

The (uniform) charge density is given by:

$$\rho = \frac{Q}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{3Q}{4\pi a^3}$$

Outside the sphere: choose a Gaussian surface of radius r, centered on the sphere of charge. On this Gaussian Surface, $\vec{A} = A\hat{r} : |\vec{E}| = \text{constant} : \vec{E} = |\vec{E}|\hat{r}$.

On the Gaussian surface, $\Phi_E = \sum_{\Delta A_i} \vec{E}_i \cdot \Delta \vec{A}_i = \sum_{\Delta A_i} E \Delta A_i = E \sum_{\Delta A_i} \Delta A_i = E(4\pi r^2)$

Then, from Gauss's law we have $\Phi_E = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow E_{out} = \frac{Q}{4\pi\epsilon_0 r^2} \Rightarrow \vec{E}_{out} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$.

Inside the sphere: choose a Gaussian surface of radius r, centered on the sphere of charge. On this Gaussian Surface, $\vec{A} = A\hat{r} : |\vec{E}| = \text{constant} : \vec{E} = |\vec{E}|\hat{r}$.

Inside the sphere, you are not enclosing all the charge; instead you only enclose charge up to a radius r. Since the charge is uniform, we can calculate the enclosed charge as:

$$Q_{enc} = \rho \, x \, \text{Volume} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{3Q}{4\pi a^3}\left(\frac{4}{3}\pi r^3\right) = Q\left(\frac{r}{a}\right)^{3.}$$

Now apply Gauss's law. Again, as above:

$$\Phi_E = \sum_{\Delta A_i} \vec{E}_i \cdot \Delta \vec{A}_i = \sum_{\Delta A_i} E \Delta A_i = E \sum_{\Delta A_i} \Delta A_i = E(4\pi r^2)$$

So: $E_{in}(4\pi r^2) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \Rightarrow E_{in} = \frac{Q}{4\pi\epsilon_0}\frac{r}{a^3} \Rightarrow \vec{E}_{in} = \frac{Q}{4\pi\epsilon_0}\frac{r}{a^3}\hat{r}$

Notice at the surface, the inside solution is equal to the outside solution.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

The surface charge density is given by $\sigma = \frac{Q}{A} = 1\frac{\mu C}{m^2}$.

Outside the capacitor note that the electric field is zero.
Choosing a cylindrical Gaussian surface of area A' we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$EA' = \frac{\sigma A'}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0} = \frac{1x10^{-6}C}{8.85x10^{-12}m^2} = 1.13x10^5\frac{N}{C}$$.