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(1) A thin converging lens has a focal length of 10 cm while a thin diverging lens has a focal length of 20 cm. The two lenses are in direct contact. If an object is located at a position of 10 cm in front (thus, a real object) of the lens combination, characterize the resulting image.

(2) Suppose you have a converging lens [L1] of focal length +f and a diverging lens [L2] of focal length –f which are separated by a distance equal to the magnitude of the focal length. If an object is placed in front of [L1] at a distance equal to ½ the focal length, characterize the resulting image.

(3) Suppose you have a converging lens [L1] and a diverging lens [L2] and the two lenses are separated by a distance d. Does it matter in an optical system whether the light from an object passes through [L1] first or [L2] first?

(4) This problem is similar to what you will do in lab. It illustrates one way to find the focal length of a diverging lens.

Consider a converging lens [L1] with a focal length f_1. A diverging lens [L2] with a focal length f_2 is used together with the first lens. [L2] is a distance d behind [L1].

(a) What is the minimum object distance s_1 that a real object for the first lens can have in order to produce a virtual object for the second lens?

(b) What is the image position in the example used in (a) when s=0.75 cm and f_2=-3 cm (d=1 and f_1=0.5)?

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(1) A thin converging lens has a focal length of 10 cm while a thin diverging lens has a focal length of 20 cm. The two lenses are in direct contact. If an object is located at a position of 10 cm in front (thus, a real object) of the lens combination, characterize the resulting image.

First, find the effective focal length since the lenses are in direct contact.

$$\frac{1}{f_{eff}}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{+10}+\frac{1}{-20}=\frac{2-1}{20}=\frac{+1}{20}\Rightarrow f_{eff}=20\,cm$$

The thin lens equation then gives:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f_{eff}}\Rightarrow\frac{1}{s'}=\frac{1}{+20}-\frac{1}{10}=\frac{1-2}{20}=-\frac{1}{20}\Rightarrow s'=-20\,cm:M=-\frac{s'}{s}=-\frac{-20}{10}=+2$$

The ultimate image is [Virtual : Upright : Magnified]

(2) Suppose you have a converging lens [L1] of focal length +f and a diverging lens [L2] of focal length –f which are separated by a distance equal to the magnitude of the focal length. If an object is placed in front of [L1] at a distance equal to ½ the focal length, characterize the resulting image.

You should not use the effective focal length here since the two lenses are not in direct contact. According to the thin lens equation:

$$\frac{1}{s'_1}=\frac{1}{f}-\frac{2}{f}=\frac{1-2}{f}=\frac{-1}{f}\Rightarrow s'_1=-f:M_1=\frac{-s'_1}{s_1}=-\frac{-f}{1/2\,f}=+2$$

Since f was positive here, this image is in front of [L1]. Now the distance to the second lens [L2] is also f. This gives an object distance for the second lens which is $s_2=+2f$. This object will be real since it is in front of the second lens. We can now find the final image characterization:

$$\frac{1}{s_2}+\frac{1}{s'_2}=\frac{1}{-f}\Rightarrow\frac{1}{s'_2}=\frac{1}{-f}-\frac{1}{s_2}=-\frac{1}{f}-\frac{1}{2f}=-\frac{2+1}{2f}=-\frac{3}{2f}\Rightarrow s'_2=-\frac{2}{3}f:M=-\frac{s'_2}{s_2}=-\frac{-2/3\,f}{2f}=+\frac{1}{3}\quad .$$

This image is a distance of (2/3)f in front of [L2] which is in turn a distance of (1/3)f behind [L1].

The second image is virtual since the image distance is less than zero (it is in front of [L2]). The magnification is found by multiplying the individual magnifications:

$M=M_1M_2=(+2)\left(\frac{+1}{3}\right)=+\left(\frac{2}{3}\right)$ .The image is thus [Virtual : Upright: reduced]

(3) Suppose you have a converging lens [L1] and a diverging lens [L2] and the two lenses are separated by a distance d. Does it matter in an optical system whether the light from an object passes through [L1] first or [L2] first?

I'll answer this question using a rather specific example: suppose that [L1] has a focal length of 1 cm while [L2] has a focal length of -1 cm. Let the lenses be separated by ½ cm. Let's place an object at a distance of 2 cm in front of the lenses in each case. From the first situation

[object:[L1]:d:[L2]]: $\frac{1}{s'_1}=\frac{1}{+1}-\frac{1}{2}=\frac{2-1}{2}=+\frac{1}{2}\Rightarrow s'_1=+2 : M_1=-\frac{s'_1}{s_1}=-\frac{+2}{2}=-1$. This (real) image is then 1.5 cm behind [L2] and thus $s_2=-3/2\,\text{cm}$ (virtual object).

The second image will be at:

$$\frac{1}{s'_2}=\frac{1}{-1}-\frac{1}{-3/2}=-1+\frac{2}{3}=\frac{-3+2}{3}=\frac{-1}{3}\Rightarrow s'_2=-3 : M_2=-\frac{-3}{-3/2}=-2$$.

The overall magnification is then $M=+2$.

Now let's work the problem the other way around:

$$\frac{1}{s'_1}=\frac{1}{-1}-\frac{1}{2}=\frac{-2-1}{2}=-\frac{3}{2}\Rightarrow s'_1=-\frac{2}{3} : M_1=-\frac{-2/2}{2}=+\frac{1}{3}$$

Since this image is virtual, it will be a distance of

$s_2=\frac{2}{3}+\frac{1}{2}=\frac{4+3}{6}=\frac{7}{6}\text{cm}$ in front of the second lens. It will also be a real object for the second lens. The second image position will be given by:

$$\frac{1}{s'_2}=\frac{1}{+1}-\frac{6}{7}=+\frac{1}{7}\Rightarrow s'_2=+7 : M=-\frac{s'_2}{s_2}=-\frac{+7}{\left(\frac{7}{6}\right)}=-6$$

The overall magnification here is then -2 : $M=M_1M_2=\frac{1}{3}(-6)=-2$ which means that it is not the same either way. (This is one of those cases where you can't say 'or the other way around).Notice that if the distance is zero (the lenses are in contact) then it does not matter which arrangement of lenses that you have. I guess this is the easy way to see that it also does not matter which way a lens is turned so long as it is a thin lens.

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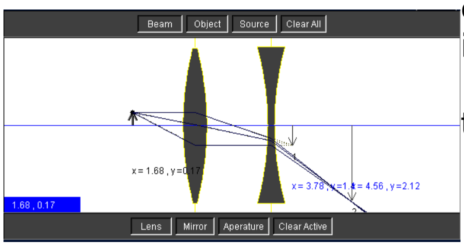
(4) This problem is similar to what you will do in lab. It illustrates one way to find the focal length of a diverging lens.

Consider a converging lens [L1] with a focal length f_1. A diverging lens [L2] with a focal length f_2 is used together with the first lens. [L2] is a distance d behind [L1].

(a) What is the minimum object distance s_1 that a real object for the first lens can have in order to produce a virtual object for the second lens?

The thin lens equation for the first lens is: $\frac{1}{s_1}+\frac{1}{s'_1}=\frac{1}{f_1}$ .In order for the image from [L1] to be a virtual object for [L2], it needs to (a) be real {since it need to be behind [L1]} and (b) be a distance >=d {since it needs to be behind [L2]}. Now you know that only objects at a greater distance than the focal length produce real images for a converging lens. That means then that at the very minimum, the object needs to be at a distance greater than the focal length. You will also need, however, that the distance is such that the image distance is less than d. Since the image does not form at the focal point, and moving the object further from the converging lens than the focal point moves the image inward towards the lens, we can obtain our bounds on where the object can be placed:

$\frac{1}{s}=\frac{1}{f}-\frac{1}{d}=\frac{d-f}{fd}\Rightarrow s=\frac{fd}{d-f}$. The bounds on f are then: $\left[f\leq s\leq\frac{fd}{d-f}\right]$. This is independent of the focal length of the diverging lens. For example, suppose f=0.5 cm and d=1 cm. You will only be able to get real images from the system if $[.5<s<1]$. Here is a capture from the optical bench that shows this situation.

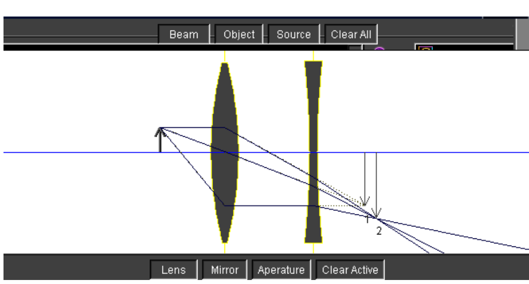


(b) What is the image position in the example used in (a) when s=0.75 cm and f_2=-3 cm (d=1 and f_1=0.5)? In this case, the first image is located at

$$\frac{1}{s'_1}=\frac{1}{f}-\frac{1}{s_1}=\frac{1}{\left(\frac{1}{2}\right)}-\frac{1}{\left(\frac{3}{4}\right)}=\frac{2}{1}-\frac{4}{3}=\frac{6-4}{3}=\frac{2}{3}\Rightarrow s'_1=+\frac{3}{2}:M_1=-\frac{+\frac{3}{2}}{-\frac{1}{2}}=-\frac{3}{2}\frac{4}{3}=-2$$

Now this is ½ cm behind the second lens. Thus, s_2=-.5 cm. We then find:

$$\frac{1}{s'_2}=\frac{1}{-3}-\frac{1}{-\frac{1}{2}}=-\frac{1}{3}+\frac{2}{1}=\frac{-1+6}{3}=+\frac{5}{3}\Rightarrow s'_2=+\frac{3}{5}:M_2=-\frac{\frac{+3}{5}}{-\frac{1}{2}}=\frac{3}{5}\frac{2}{1}=\frac{6}{5}$$



The overall magnification is

$$M=M_1M_2=-2\left(\frac{6}{5}\right)=\frac{-12}{5}=-2.4$$

The ultimate image is [Real : Inverted : Magnified]