	Physics 220: Worksheet 20	Name: ________________

(1) A lens produces a virtual image of an object which is upright and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

(2) A lens produces a real image of an object which is inverted and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

(3) Some examples of virtual objects:
Suppose a virtual object is located 10 cm behind a diverging lens with a focal length of 5 cm. Characterize the resulting image.

(4) Suppose a virtual object is located 10 cm behind a converging lens with a focal length of 5 cm. Characterize the resulting image.

(5) Where must an object be located if a real image at 10 cm results from a diverging lens with a focal length of 15 cm? Characterize the resulting image if the object is upright.

(6) An object is at a distance s_0 from a converging lens of focal length f (s_o>f). The object is moving towards the lens with a speed v. How fast is the point at which the image would form moving at all times?

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(1) A lens produces a virtual image of an object which is upright and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

$$M = \frac{h'}{h} = +2 = -\frac{s'}{s} = -\frac{s'}{+20} \Rightarrow s' = -40$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{+20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40} = \frac{1}{f} \Rightarrow f = +40\,\mathrm{cm}$$

Notice that it's not possible here to have a real image in this situation.

(2) A lens produces a real image of an object which is inverted and 2 cm high when a 1 cm high object is placed 20 cm from the lens. What is the focal length of the lens and what is the image position?

$$M = \frac{h'}{h} = -2 = -\frac{s'}{s} = -\frac{s'}{+20} \Rightarrow s' = +40$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{+20} + \frac{1}{+40} = \frac{2+1}{40} = \frac{3}{40} = \frac{1}{f} \Rightarrow f = +\frac{40}{3} = +13.3\,\mathrm{cm}$$

(3) Some examples of virtual objects:
Suppose a virtual object is located 10 cm behind a diverging lens with a focal length of 5 cm. Characterize the resulting image.

Since the object is virtual, we have $s = -10\,\mathrm{cm}$. This could have been created as the result of a second lens. Now we can find the image position from the thin lens equation:

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-5} - \frac{1}{-10} = -\frac{2}{10} + \frac{1}{10} = -\frac{1}{10} \Rightarrow s' = -10\,\mathrm{cm}$$

$$M = -\frac{s'}{s} = -\frac{-10}{-10} = -1$$

The image is : $[\mathrm{virtual} : \mathrm{unmagnified} : \mathrm{inverted}]$

(4) Suppose a virtual object is located 10 cm behind a converging lens with a focal length of 5 cm. Characterize the resulting image.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{+5} - \frac{1}{-10} = \frac{2}{10} + \frac{1}{10} = +\frac{3}{10} \Rightarrow s' = +\frac{10}{3} = +3.33\,\mathrm{cm}$$

$$M = -\frac{s'}{s} = -\frac{+3.33}{-10} = +0.333$$

The image is : $[\mathrm{real} : \mathrm{reduced} : \mathrm{upright}]$

(5) Where must an object be located if a real image at 10 cm results from a diverging lens with a focal length of 15 cm? Characterize the resulting image if the object is upright.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{-15} - \frac{1}{+10} = \frac{-2-3}{30} = -\frac{5}{30} = -\frac{1}{6} \Rightarrow s = -6\,\mathrm{cm}$$

$$M = -\frac{s'}{s} = -\frac{+10}{-6} = +1.67$$

The image is: $[\mathrm{real} : \mathrm{enlarged} : \mathrm{upright}]$

(6) An object is at a distance s_0 from a converging lens of focal length f (s_o>f). The object is moving towards the lens with a speed v. How fast is the point at which the image would form moving at all times?

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s_0-vt-f}{f(s_0-vt)}\Rightarrow s'=\frac{f(s_0-vt)}{s_0-vt-f}$$

The speed of the image is given by:

$$v'=\frac{\Delta s'}{\Delta t}=\frac{f(-v)}{s_0-vt-f}-\frac{f(s_0-vt)}{[s_0-vt-f]^2}(-v)=-\frac{fv}{[s_0-vt-f]}\left\{1-\frac{s_0-vt}{s_0-vt-f}\right\}=-\frac{fv}{[s_0-vt-f]}\left\{\frac{s_0-vt-f-s_0+vt}{[s_0-vt-f]}\right\}=\frac{f^2v}{[s_0-vt-f]^2}$$

At one point, this speed will be infinite. There is not really any information contained in this fact so it does not violate relativity. I have, however, been very careful with my words in the statement of this problem. When does the speed become infinite? This is when

$$s_0-vt-f=0\Rightarrow vt=s_0-f\Rightarrow t=\frac{s_0-f}{v}$$

What is the object location at this time?

$$s=s_0-vt=s_0-v\left[\frac{s_0-f}{v}\right]=s-s_0+f=f\ .$$

When the object reaches the focal length, the point at which the image forms would move infinitely fast (it would take light a while to catch up with this point).