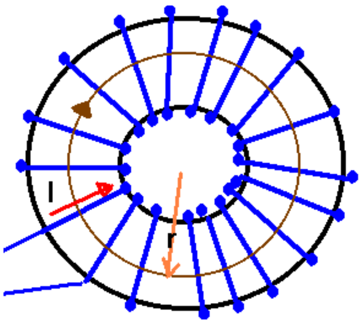
2020	Physics **220**: Worksheet13	Name _______________



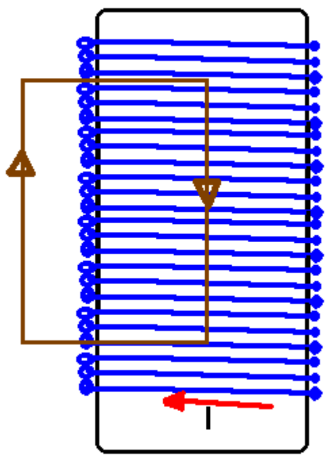
(1) Consider the torrid consisting of N turns as shown. Assume that the winding is tight enough so that edge effects are not important. Calculate the magnetic field inside the torrid from Ampere's law.

Solution: On the brown path, we have (from the right hand rule) that B circulates around as shown. The windings are assumed to be close enough so that B does not really come out of the inside of the torrid. This is usually reinforced by making the torrid out of a ferrite which serves to trap the magnetic field lines. Iron is an example of such a material. On the (brown) path shown, then we have:

$$\sum_{\text{curve}} \vec{B}_i \cdot \Delta \vec{s}_i = B \sum_{\text{curve}} \Delta s_i = B(2\pi r) \quad \text{(Non-calculus)}$$

According to Ampere's law then $B(2\pi r)=\mu_0 I_c$. Here, $I_c = NI$ where I is the current injected into the torrid and N is the number of turns on the torrid. Thus, solving for B we find:

$$\vec{B}=\frac{\mu_0 NI}{2\pi r}(-\hat{\theta}) \quad .$$



(2) Consider the solenoid of length L with a number of turns per unit length n. Find the magnetic field in regions near the center of the solenoid.

Solution: Consider the (brown) path shown. On the path, if the winding is tight enough, as we shall assume here, then B inside is parallel to the path. Outside the solenoid, near the center length wise, the magnetic field lines are spread out enough so that the field is nearly zero which is an appropriate approximation here. On the sides, B and S are perpendicular. Let the path have dimensions a (width) and b(length). Then, we need to evaluate:

$$\sum_{\text{curve}} \vec{B}_i \cdot \Delta \vec{s}_i = B \sum_{\text{curve}} \Delta s_i = B(w) \quad \text{(Non-calculus)}$$

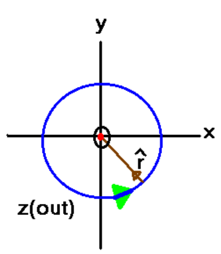
The current enclosed by this path is then given by $I_c = (nw) I$ where I is the current injected into the solenoid. Thus, we can solve for the magnetic field to obtain: $\vec{B} = \mu_0 n I \hat{z}$ where the z direction is towards the top of the paper. It is possible to show with a more detailed analysis that near the ends of the solenoid, due to field line leakage (approximately) that the magnetic field is 1/2 of this value. You can understand this by thinking that at any given slice, the magnetic field comes in part from the solenoid near by and above and the other part comes from below. If one of these components is missing, you'd expect a reduction of about 1/2 in this value. The solenoid is also very useful for producing uniform magnetic fields and ***is very important for you to 100% make sure you understand since it will play a similar role for magnetic fields that a parallel plate capacitor played for electrostatic problems.***

(3) The law of Biot-Savart. This is going to turn out to be a different form of Ampere's law. For non-calculus students, we can approximately write it as follows ... imagine a small element of wire of length $\Delta\vec{S}$ which is containing (or, guiding if you prefer) a current I. This small *current element* gives rise to a magnet field which I will denote by $\delta\vec{B}$ and if you add up (vectorially) all these small magnetic fields, you get the total magnetic field ... or $\vec{B}=\sum\delta\vec{B}$ where it is emphasized that this is a vector sum which means that you need to add up the components of B. In any event, with this introduction, the non-calculus version of the law of Biot-Savart would appear as:

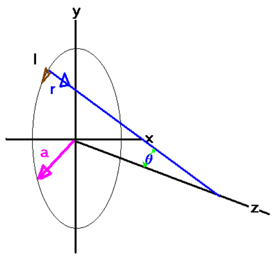
$$\delta\vec{B}=\left(\frac{\mu_0}{4\pi}\right)\frac{I\Delta\vec{S}\times\hat{r}_{ip}}{\vec{r}_{ip}^{\,2}}$$

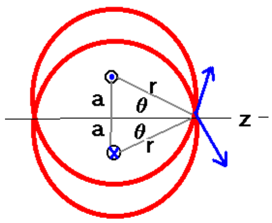
where r is the vector directed from the current element to a point in space and $\vec{r}_{ip}$ is the unit vector directed to this point from the current element in question. Whew! In general, there is nothing that prevents non-calculus students from working with this for fairly complicated shapes when given a good computer... in practice, really the only time you can do it easily without calculus turns out to be for the very important case of calculating the magnetic field along the axis of a circular current loop.

Here, $\vec{r}_{ip}$ is the vector directed from the current element to a point in space and $\hat{r}_{ip}$ is the unit vector directed to this point from the current element in question.



Let's calculate the magnetic field first at the center of a circular current loop of radius a carrying a current I. If the current flows as shown by the blue angle, then B is out of the paper as shown. In this case, each element of the wire causes a magnetic field to be in the same direction, along the +Z direction. The angle between each element and r is exactly 90^0 so the calculation reduces to the following: $\delta\vec{B}=\left(\frac{\mu_0}{4\pi}\right)\frac{I\Delta S}{r^2}\hat{z}$. Now if you add up ΔS over the circumference of a circle, what you have done is to answer the question "what is the circumference of a circle" which is $2\pi r$. In this case, r=a and so we have the result that the vector magnetic field is given by $\vec{B}=\left(\frac{\mu_0}{2}\right)\frac{I}{a}\hat{z}$. For yucks, let's write this in terms of the magnetic moment which we defined in the last lecture as $\vec{\mu}=I\vec{A}$ ... since for the circle A is given by πa^2, we then find the result $\vec{B}=\frac{\vec{\mu}\mu_0}{2\pi a^3}$. At best, this is a very loose argument for the non-calculus students. It is probably best to remember the result here.





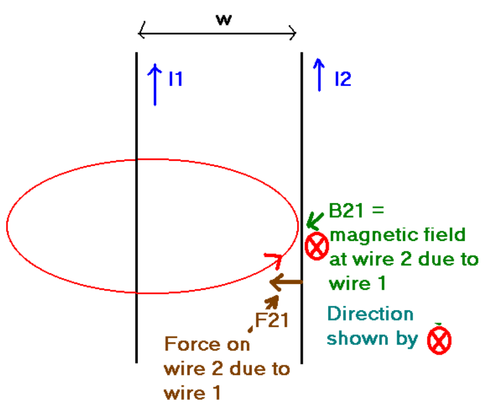
(4) Calculate the magnetic field along the symmetry axis of a circular current loop of radius a, carrying a current I.

Solution: The particular geometry of the system is shown to the left. We are interested in calculating the magnetic field along the z axis. We need to find the direction of the magnetic field at a point along the z-axis from a small current element. You might imagine that the high degree of symmetry is important here, which it is. Now, refer to the second image. This shows the magnetic fields arising at some point along z due to upper and lower currents (one of which is directed out of the page, the other of which is directed into the page). The off-axis directions of these magnetic fields will cancel when added up so that the only components of the magnetic fields which will survive are those components along the z-direction which will be given by $B\sin(\theta)$ (the angle between B and z is 90^0-θ which gives $\sin(\theta)$). We can then easily put it all together once we realize that $\sin(\theta)=\frac{a}{r}$ and $r=\sqrt{z^2+a^2}$. Thus, we have the result:

$$\vec{B}=\left(\frac{\mu_0 I}{4\pi}\right)\frac{2\pi a^2}{\left[z^2+a^2\right]^{3/2}}\hat{z}=\left(\frac{|\vec{\mu}|\mu_0}{2\pi}\right)\left[z^2+a^2\right]^{3/2}\hat{z} \quad .$$

Notice that for z=0, the result becomes exactly the same result as we had before.

(5) Consider two long wires, each carrying a current I. Find the magnitude of the force of attraction (or repulsion) between the two wires. Then, find out if the force is attractive or repulsive.



Solution:

Consider the situation shown. The force on wire 2 due to wire 1 is given by $F_{21}=I_2 L_2 B_{21}$. From Ampere's law, $B_{21}=\frac{\mu_0 I_1}{2\pi w}$ so the magnitude of F_{21} is given by $|\vec{F}_{21}|=\frac{\mu_0 I_1 I_2}{2\pi w}L_2$ and the force per unit length on wire 2 is given by $\left|\frac{\vec{F}_{21}}{L_2}\right|=\frac{\mu_0 I_1 I_2}{2\pi w}$ The force as shown is towards wire 1. By Newton's 3rd law, we then have that the force per unit length on wire 1 is the same but in the opposite direction. Thus if currents are in the same direction, the force is attractive. If the currents are in opposite directions, it is now easy to show (use RHR#1 but let I_2 go in the opposite direction) that the force would be repulsive.