

Physics 250: UnQuiz 02

A sphere of radius b has a total charge $-Q$. Inside at the center is a charge $+2Q$. The sphere otherwise has a uniform charge density. Find the vector electric field for $r < b$ and $r > b$.

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Gauss's law says: $\Phi_E = \frac{Q_{\text{enclosed}}}{\epsilon_0}$ Choose a Gaussian surface centered on the sphere. We need to calculate the volume charge density.

$$\text{For the sphere: } \rho = \frac{Q}{\text{volume}} = \frac{-Q}{\frac{4}{3}\pi b^3}$$

On the Gaussian Surface, (1) $\vec{E} = |\vec{E}|\hat{r}$, $\vec{E} \parallel \vec{A}$ and (2) $|\vec{E}| = \text{constant}$.

$$(1) \quad r \leq b : \quad Q_{\text{enclosed}} = 2Q + \frac{4}{3}\pi r^3(\rho) \quad \Phi_E = \oiint \vec{E} \cdot d\vec{A} = \oiint E dA = E \oiint dA = E(4\pi r^2)$$

$$E_{r \leq b}(4\pi r^2) = \frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} + \frac{2Q}{\epsilon_0} \Rightarrow \vec{E}_{r \leq b} = \frac{2Q}{4\pi\epsilon_0 r^2} + \frac{r\rho}{\epsilon_0} = \frac{Q}{2\pi\epsilon_0 r^2} - \frac{rQ}{\frac{4}{3}\pi b^3 \epsilon_0} = \frac{Q}{2\pi\epsilon_0 r^2} - \frac{3rQ}{4\pi\epsilon_0 b^3}$$

$$\vec{E}_{r \leq b} = \left[\frac{Q}{2\pi\epsilon_0 r^2} - \frac{3rQ}{4\pi\epsilon_0 b^3} \right] \hat{r}$$

(2) $r \geq b$: In the case all the charge is enclosed. Thus the total charge enclosed is given by : $-Q + 2Q = Q$.

On the Gaussian Surface, (1) $\vec{E} = |\vec{E}|\hat{r}$, $\vec{E} \parallel \vec{A}$ and (2) $|\vec{E}| = \text{constant}$.

$$Q_{\text{enclosed}} = Q \quad \Phi_E = \oiint \vec{E} \cdot d\vec{A} = \oiint E dA = E \oiint dA = E(4\pi r^2)$$

$$E_{r \geq b}(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow E_{r \geq b} = \frac{Q}{4\pi\epsilon_0 r^2} \Rightarrow \vec{E}_{r \geq b} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$