2.3.5 **Electrostatic boundary conditions when crossing a surface charge σ**

Solve the Gauss's law for an infinite plane of surface charge σ, making a sketch and noting the image in the text ( Figure 2.36 on page 88).

$$\vec{E}_{below} = -\vec{E}_{above}$$

From the Gauss's law solution, we have the results, in fact:

$$\vec{E}_{below} = \frac{-\sigma}{2\epsilon_0}\hat{z} : \vec{E}_{above} = \frac{\sigma}{2\epsilon_0}\hat{z}$$

So the discontinuity is :

$$\vec{E}_{above} - \vec{E}_{below} = \left(\frac{\sigma}{2\epsilon_0} - \frac{-\sigma}{2\epsilon_0}\right)\hat{z} = \frac{\sigma}{\epsilon_0}\hat{z}$$

Since the electric field is only in the z direction, in fact we call this the “normal” E or, the boundary condition for the electric field is:

$$\triangle E_{\perp} = \frac{\sigma}{\epsilon_0}$$

Upon crossing a surface the tangential component of E is continuous.

$$\triangle E_{\parallel} = 0$$

Even if on one side, the electric field is zero.

Why? Consider a square loop as shown in figure 2.37 page 89 (which is correct). Then:

$$\oint_{\text{closed loop}} \vec{E}\cdot d\vec{s} = E_{\parallel\, above}\, l + 0 - E_{\parallel\, below}\, l + 0 = 0 \Rightarrow E_{\parallel\, above} = E_{\parallel\, below}$$

No bets though for other than static electric fields (and these electric fields must arise directly from charge distributions) here.

The combination of these two boundary conditions gives:

$$\vec{E}_{above} - \vec{E}_{below} = \frac{\sigma}{\epsilon_0}\hat{n}$$

We can calculate the potential difference upon crossing a boundary:
Take a perpendicular path from z=-a to z=+1 through the plane:

$$V_{above} - V_{below} = -\oint \vec{E} \cdot d\vec{s} = \frac{-\sigma}{\epsilon_0} a + \frac{\sigma}{\epsilon_0} a = 0 \Rightarrow V_{above} = V_{below}$$

The potential is continuous. Since, however, the electric field comes from the gradient of V, the discontinuity in the gradient of V is:

$$\vec{\nabla} V_{above} - \vec{\nabla} V_{below} = \frac{-\sigma}{\epsilon_0} \hat{n}$$

This can also be written as:

$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = \frac{-\sigma}{\epsilon_0}$$

where the normal derivative is given by:

$$\frac{\partial V}{\partial n} \equiv \vec{\nabla} V \cdot \hat{n}$$