

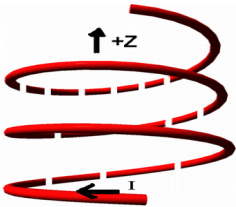
Instructions: You have a total of 50 minutes to complete this test.

Answer each of the following questions completely, showing full details with correct SI units.

Time Start _____ Time finish _____ Pledged _____

Do not discuss any aspect of this test with anyone until I return the test.

Constants: $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$; $\mu \equiv 1 \times 10^{-6}$



[1] An ideal solenoid has a total length h and the interior cross sectional area is A with windings as shown to the right. A current I is injected into the solenoid at the bottom and exits at the top as shown. Note that in the image to the right, dashed portions are behind while solid portions are in front.

[a] Circle the correct direction for the magnetic field inside the solenoid.

$+\hat{z}$ $-\hat{z}$

In answering the following questions, you must show complete details leading up to your answer for full credit.

[b] Calculate the magnitude of the magnetic field inside the solenoid near the center. **You must show details, use words, include assumptions and sketches. Your answer involves n , I and a constant.**

[c] Suppose the solenoid has a total of N turns. Assuming the magnetic field is uniform throughout the solenoid, calculate the inductance of the solenoid. Your answer involves n , and the interior volume of the solenoid

[d] Calculate the magnetostatic energy density of the solenoid. Here, express your answer in terms of a constant and B .

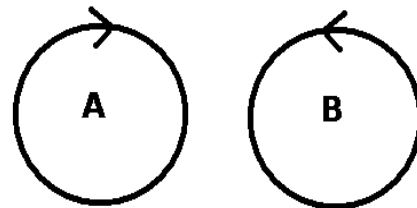
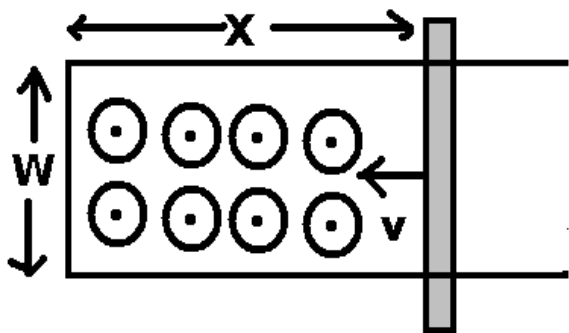
[e] Provide numerical answers for (b), (c), and (d) **together with correct SI units** for the case $I=10\text{A}$, $n=4000/\text{m}$, $A=0.1 \text{ m}^2$, $h=1 \text{ m}$.

[e:b] _____

[e:c] _____

[e:d] _____

[2] Consider the following situation: conducting rail is moving with an instantaneous position given by $x = x_0 - pt^4$ (x_0 and p are constants and the rail is moving in the $-x$ direction). In the area enclosed by the rail system, a uniform magnetic field (B) is directed into the positive z direction: $\vec{B} = |\vec{B}|\hat{z}$. The rail system has a total resistance (at the end only) given by R and this value is assumed to be constant throughout this problem.



[a] At an instant in time, calculate the magnetic flux through the enclosed region of the system in terms of B , x and w . You may assume the normal to the area of the enclosed region points in the $+z$ direction.

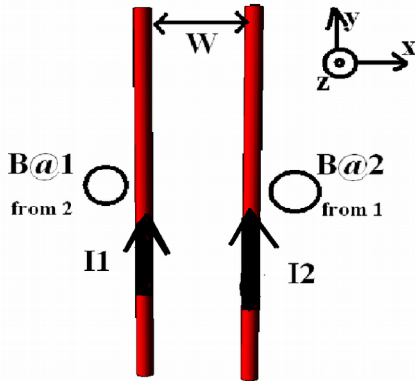
[b] Find the magnitude of the induced emf in the system at any time t in terms of B , W , p and t .

[c] Which direction will the induced current flow: (A or B) and why (in words). Be very clear in your answer to this question.

[d] Suppose $p = 5 \text{ m/s}^4$, $w = 0.5 \text{ m}$, $x_0 = 100 \text{ m}$, $t = 2 \text{ s}$, and $B = 0.22 \text{ T}$. Provide numerical answers to [a] and [b] **together with correct SI units**.

[d:a] _____

[d:b] _____



[3] Two long wires (each with the same length h) carry currents I_1 and I_2 in the directions shown. In the coordinate system indicated, z is out of the paper. You may assume the wires are long enough so that they may be treated as ideal. **Note that a dot represents a vector coming out of the page while a cross is a vector pointing into the page.**

[a] In the circles provided, show the direction that the magnetic fields point for B at 1 from 2 and B at 2 from 1. Use these symbols: \odot if the field is in the $+z$ direction and \otimes for the field directed into the $-z$ direction.

[b] Showing **complete details**, calculate the magnitude of the magnetic field B at 1 from 2.

[c] Calculate the **vector force** on wire 1 due to wire 2. **You must include correct unit vectors here for credit.**

[d] Suppose that $I_1=3A$, $I_2=3A$, $w=0.5$ m and $h=3.0$ m. Provide numerical answers for [b] and [c] **together with correct SI units.**

[d:b] _____

[d:c] _____

[4] A transformer consists of two coils with $N_p=20$ turns on the primary side and $N_s=10$ turns on the secondary side. You may assume perfect flux coupling here. **Be sure to use correct SI units in your answers.**

[a] If a DC voltage of 10 V is input on the primary side of the transformer, what is the output voltage?

[b] If an AC voltage of 100 V (RMS) is input on the primary side of the transformer, what is the (RMS) output voltage?

[c] Suppose a circuit consists of an inductor ($L=2\mu\text{H}$), and a capacitor ($C=2\mu\text{F}$) in a **series configuration**. Calculate the resonance frequency (f_0) of this circuit.

[d] Calculate the impedance (Z) of the circuit described in part [c] if it also has a resistance in series given by $R=100\Omega$. when operated at a frequency of $f=60\text{Hz}$. Be sure to provide correct SI units here.

$Z=$ _____