	Physics **250**: Worksheet 3A	Name: _______________

Supplementary problems for electrostatics

2016

1. Three identical charges each with $q=1\mu C$ are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the center of the triangle.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge density which varies as $\rho=\rho_0\frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge $Q=1\mu C$ on one plate and $Q=-1\mu C$ on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

	Physics **250**: Worksheet 3A	Name: ________________

1. Three identical charges each with $q=1\mu C$ are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the center of the triangle.

The hard part here is to locate coordinates of the points. Let the triangle have a side of length L. Locate one point at the origin. Locate the second point at (L,0). We need to find the coordinates of the center, and the 3rd point. The x coordinate of the 3rd point is x=L/2. The y coordinate is then given by:

$$L^2=\left(\frac{L}{2}\right)^2+y^2 \Rightarrow y^2=L^2-\frac{L^2}{4}=3\frac{L^2}{4} \Rightarrow y=\frac{\sqrt{3}}{2}L\ .$$

The coordinates of the center of the triangle are then:

$$x=\left(\frac{L}{2},\frac{\sqrt{3}}{4}L\right)\ .$$

In our present case, L=1 m. We thus have the coordinates for this particular orientation:

$$1:(1\mu;0,0),2:(1\mu,1,0),3:(1:.5,\frac{\sqrt{3}}{2}),p:(0.5,\frac{\sqrt{3}}{4})$$

Calculate the following vectors: $\vec{r}_{1,}\vec{r}_{2,}\vec{r}_{3,}\vec{r}_p,\vec{r}_{1p},\vec{r}_{2p},\vec{r}_{3p}$. These vectors are:

$$\vec{r}_p=0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_1=0\,\hat{x}+0\,\hat{y}:\vec{r}_{1p}=\vec{r}_p-\vec{r}_1=0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}-(0\,\hat{x}+0\,\hat{y})=0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_2=1\,\hat{x}+0\,\hat{y}:\vec{r}_{2p}=\vec{r}_p-\vec{r}_i=\left(0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-(1\,\hat{x}+0\,\hat{y})=-0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_3=0.5\,\hat{x}+\frac{\sqrt{3}}{2}\hat{y}:\vec{r}_{3p}=\vec{r}_p-\vec{r}_i=\left(0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-\left(0.5\,\hat{x}+\frac{\sqrt{3}}{2}\hat{y}\right)=0\,\hat{x}-\frac{\sqrt{3}}{4}\hat{y}$$

Now write E directly.

$$\vec{E}_p=k\,\mu\left[\hat{x}(0.5-0.5+0)+\hat{y}\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}\right)\right]=k\,\mu\left[0\,\hat{x}-\frac{\sqrt{3}}{4}\hat{y}\right]$$

$$\Rightarrow\vec{E}_p=[0\,\hat{x}-3892\,\hat{y}]\frac{N}{C}$$

	Physics **250**: Worksheet 3A	Name: ________________

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

Notice that if $\vec{r}_p = 0\hat{x} + 0\hat{y}$, then $\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = \vec{0} - \vec{r}_i = -\vec{r}_i$. We can use this to write down the electric field directly:

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+1\hat{y})}{[1^2+1^2]^{3/2}} + \frac{2(-1\hat{x}+1\hat{y})}{[2]^{3/2}} + \frac{3(-1\hat{x}-1\hat{y})}{2\sqrt{2}} + \frac{4(\hat{x}-\hat{y})}{2\sqrt{2}}\right]$$

$$\Rightarrow \vec{E}_p = \frac{k\mu}{2\sqrt{2}}[\hat{x}(1-2-3+4)+\hat{y}(1+2-3-4)] = \frac{k\mu}{2\sqrt{2}}(-4\hat{y}) = \frac{-2k\mu}{\sqrt{2}}\hat{y} = -\sqrt{2}k\mu\,\hat{y} = -12713\hat{y}\,\frac{N}{C}$$

3. Three charges are arranged as follows:(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

This satisfies the condition $\vec{r}_p = \vec{0}$. We can then write the field directly.

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+0\hat{y})}{[1^2+0^2]^{3/2}} + \frac{-1(-1\hat{x}+0\hat{y})}{1} + 1(0\hat{x}-1\hat{y})\right] = k\mu[\hat{x}(1+1+0)+\hat{y}(0+0-1)] = k\mu[2\hat{x}-1\hat{y}]$$

$$\Rightarrow \vec{E}_p = [17980\hat{x} - 8990\hat{y}]\frac{N}{C}$$

	Physics **250**: Worksheet 3A	Name: ________________

4. A sphere of radius a has a charge density which varies as $\rho = \rho_0 \frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

One important point from this problem: you need to be able to integrate the charge density over a sphere. Here is the simplification if you have spherical symmetry, for a sphere which is located at the origin:

$$\oiiint \rho(r)\, d^3 r = 4\pi \int \rho(r)(r^2\, dr)$$

(1) What is the total charge Q deposited on the sphere?

$$Q = 4\pi \int_{r=0}^{r=a} \rho(r) r^2 dr = 4\pi \frac{\rho_0}{a} \int_{r=0}^{r=a} r^3 dr = \pi \frac{\rho_0}{a} a^4 = \pi \rho_0 a^3 \Rightarrow \rho_0 = \frac{Q}{\pi a^3}$$

Outside the sphere: choose a Gaussian surface of radius R, centered on the sphere of charge. On this Gaussian Surface, $\vec{A} = A\hat{r} : |\vec{E}| = \text{constant} : \vec{E} = |\vec{E}|\hat{r}$.

Gauss's law says: $\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

Calculate the flux:

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \oiint E dA = E \oiint dA = E(4\pi r^2) : Q_{enc} = Q \Rightarrow \vec{E}_{out} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = \frac{\rho_0 a^3}{4\epsilon_0 r^2}\hat{r} \quad .$$

Inside the sphere: $Q_{enc} = 4\pi \int_{r=0}^{r=r} \rho r^2 dr = 4\pi \int_{r=0}^{r=r} \rho_0 \frac{r}{a} r^2 dr = \frac{4\pi\rho_0}{a} \int_{r=0}^{r=r} r^3 dr = \frac{4\pi\rho_0}{a} \frac{r^4}{4} = \pi \frac{\rho_0}{a} r^4$.

So from Gauss's law we have (using the same Gaussian surface but now $r \leq a$):

$$E_{in}(4\pi r^2) = \frac{\pi}{\epsilon_0} \frac{\rho_0}{a} r^4 \Rightarrow \vec{E}_{in} = \frac{\rho_0 r^2}{4\epsilon_0 a}\hat{r} = \frac{Q}{\pi a^3} \frac{r^2}{4\epsilon_0 a}\hat{r} = \frac{Q}{4\pi\epsilon_0} \frac{r^2}{a^4}\hat{r}$$

Notice at the surface, the inside solution is equal to the outside solution.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

The surface charge density is given by $\sigma = \frac{Q}{A} = 1 \frac{\mu C}{m^2}$.

Outside the capacitor note that the electric field is zero.
Choosing a cylindrical Gaussian surface of area A' we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$E A' = \frac{\sigma A'}{\epsilon_0} \Rightarrow E = \frac{\sigma}{\epsilon_0} = \frac{1\text{x}10^{-6} C}{8.85\text{x}10^{-12} m^2} = 1.13\text{x}10^5 \frac{N}{C} \quad .$$