

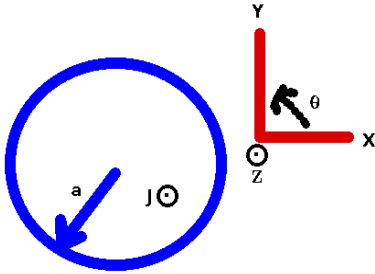
Instructions: You have a total of 50 minutes to complete this test.

Answer each question completely showing complete details.

For complete credit you must include correct SI units with numerical answers.

Time Start _____ Time finish _____ pledged _____

Constants: $k = 8.987 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$: $\epsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$: $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}$: $\mu = 1 \times 10^{-6}$



[1] A cylindrical wire of cross sectional area πa^2 has a radius a as shown. The wire carries current density given by $\vec{J} = cr^3 \hat{z}$. **Hint:** you will have to use the differential cross sectional area $dA = 2\pi r dr$ here with appropriate limits.

[a] If the wire carries a total current I , find the constant c in terms of the radius of the wire.

For the rest of the problem, use $J = cr^3$ (do not substitute your value of c obtained above).

[b] Find the **vector magnetic field** inside the wire ($r < a$) in terms of r and J (written as $J = cr^3$). Be sure to include the appropriate unit vector here.

[c] Find the vector magnetic field outside the wire ($r > a$) in terms of r , a and J (written as $J = cr^3$). Again, be sure to include the appropriate unit vector here.

[d] If $a = 2$ m, and $c = 1 \text{ A/m}^4$, provide a numerical value for the magnitude of the magnetic field at the surface of the wire (where $r = a$) with correct SI units.

[2] A parallel plate capacitor has plates of area A and separation d . On the plate located at the $z=0$, a surface charge density $+\sigma$ exists while on the plate located at $z=d$, a surface charge density $-\sigma$ exists. Answer the following assuming that this can be regarded as an ideal capacitor.

(a) Starting from Gauss's Law, show (showing complete details) how to calculate the vector electric field within the capacitor in terms of σ , ϵ_0 and \hat{z} .

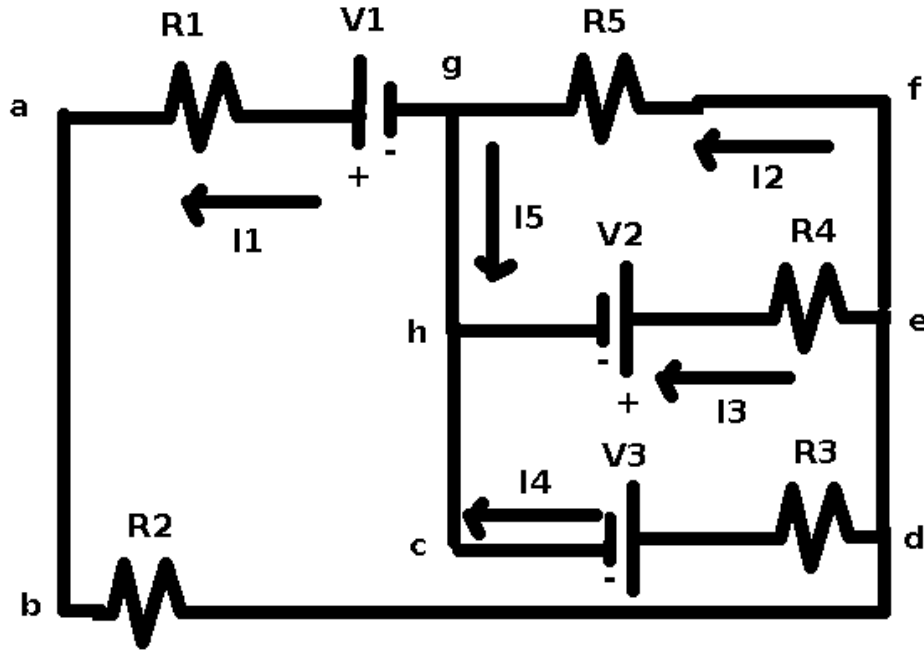
(b) Calculate the **magnitude** of the potential difference between the two plates in terms of σ , ϵ_0 and d .

(c) What is the capacitance of the capacitor in terms of ϵ_0 , A and d .

(d) What is the total energy stored on the capacitor in terms of C and V .

(e) Calculate the energy density in terms of ϵ_0 and E .

[3] Consider the circuit shown above. Write the Kirchoff's laws equations that result from the following:



Loop (abdchga) _____

Loop (ghefg): _____

Loop (hcdeh): _____

@g: _____

@h: _____

Suppose with the following values:

$V_1=10\text{V}$: $V_2=20\text{V}$: $V_3=30\text{V}$: $R_1=1\Omega$: $R_2=2\Omega$: $R_3=3\Omega$: $R_4=4\Omega$, the following currents resulted: $I_1=-2.08\text{A}$, $I_2=3.44\text{A}$, $I_3=-0.94\text{A}$, $I_4=-4.58\text{A}$, $I_5=5.52\text{A}$

Calculate the power by resistor R3 with correct SI units.

$P=$ _____

What is the interpretation of the value provided for current I_1 ?

[4] A resistor has a resistivity of $2 \Omega\text{m}$. A resistor from this material is 1 m long and has a cross sectional area of 0.1 m^2 . Be sure to provide correct SI units to your answers.

(a) Calculate the resistance of this resistor.

(b) If R_1 has a resistance of 10Ω and resistor R_2 has a resistance of 5Ω , calculate the resistance of the two resistors when connected in parallel.

(c) If R_1 has a resistance of 10Ω and resistor R_2 has a resistance of 5Ω , calculate the resistance of the two resistors when connected in series.

A capacitor has a plate area of 1.5 m^2 and a plate separation of 0.01 m .

(d) Calculate the capacitance of this capacitor.

(e) If C_1 has a capacitance of $2 \mu\text{f}$ and C_2 has a capacitance of $3 \mu\text{f}$, calculate the capacitance of the two capacitors when they are connected in parallel.

(f) If C_1 has a capacitance of $2 \mu\text{f}$ and C_2 has a capacitance of $3 \mu\text{f}$, calculate the capacitance of the two capacitors when they are connected in series.

(g) Calculate the RC time constant when a resistor ($R=1 \times 10^4 \Omega$) is connected in series with a capacitor ($C=5 \mu\text{f}$).