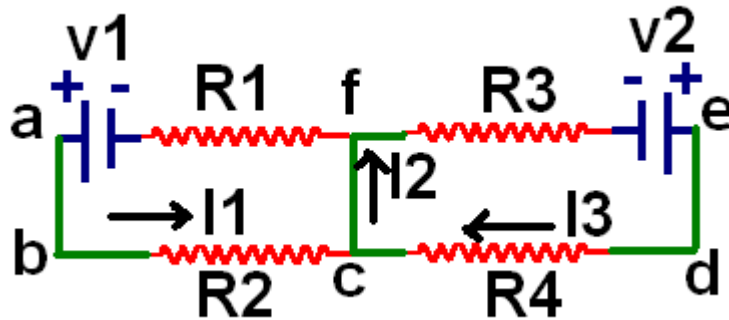


Consider the circuit shown below. Find the currents in each branch.



We have then 2 loops and 1 junction. Kirchoff's laws say:

$$(abcfa): v_1 - I_1 R_1 - I_1 R_2 = 0$$

$$(edcfe): v_2 - I_3 R_4 - I_3 R_3 = 0$$

$$(@c): I_1 + I_3 - I_2 = 0$$

$$v_1 - I_1 R_1 - I_1 R_2 = 0$$

$$v_2 - I_3 R_4 - I_3 R_3 = 0$$

$$I_1 + I_3 - I_2 = 0$$

Solutions for currents:

$$I_1 = \frac{V_1}{R_1 + R_2}; I_2 = \frac{V_1}{R_1 + R_2} + \frac{V_2}{R_3 + R_4}; I_3 = \frac{V_2}{R_3 + R_4}$$

The answer should not be too much of a surprise.

Note that at Wims, you would enter:

$$V_1 - I_1 * R_1 - I_1 * R_2 = 0$$

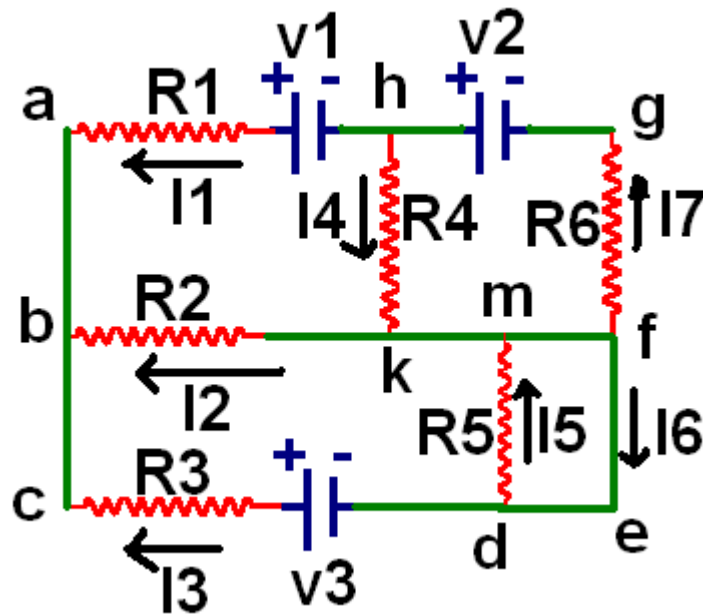
$$V_2 - I_3 * R_4 - I_3 * R_3 = 0$$

$$I_1 + I_3 - I_2 = 0$$

parameters: R1,R2, R3, R4, V1, V2

Solution:

$$\{ i_1 = v_1/(r_2+r_1), i_2 = (r_2 v_2+r_1 v_2+(r_4+r_3) v_1)/(r_2(r_4+r_3)+r_1(r_4+r_3)), i_3 = v_2/(r_4+r_3) \}$$



Needed: 4 loops, 3 junctions  
 (abkha):  $I_2 \cdot R_2 + I_4 \cdot R_4 + V_1 - I_1 \cdot R_1 = 0$   
 (bcdmb):  $I_3 \cdot R_3 - V_3 - I_5 \cdot R_5 - I_2 \cdot R_2 = 0$   
 (hkfgh):  $-I_4 \cdot R_4 - I_7 \cdot R_6 + V_2 = 0$   
 (defmd):  $I_5 \cdot R_5 = 0$   
 @h:  $I_7 - I_1 - I_4 = 0$   
 @b:  $I_1 + I_2 + I_3 = 0$   
 @kmf:  $I_5 - I_2 - I_6 + I_4 - I_7 = 0$

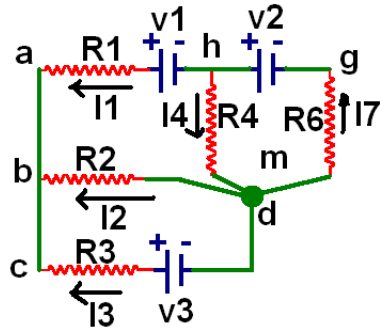
At Wims, enter:  
 $I_2 \cdot R_2 + I_4 \cdot R_4 + V_1 - I_1 \cdot R_1 = 0$   
 $I_3 \cdot R_3 - V_3 - I_5 \cdot R_5 - I_2 \cdot R_2 = 0$   
 $I_4 \cdot R_4 - I_7 \cdot R_6 + V_2 = 0$   
 $I_5 \cdot R_5 = 0$   
 $I_7 - I_1 - I_4 = 0$   
 $I_1 + I_2 + I_3 = 0$   
 $I_5 - I_2 - I_6 + I_4 - I_7 = 0$

With parameters:  $R_1, R_2, R_3, R_4, R_5, R_6, V_1, V_2, V_3$

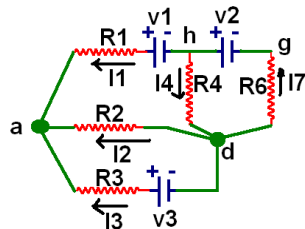
The result is a bit nasty:

$$\{ i_1 = \frac{(-r_2 r_6 v_3 + r_4(r_2(v_2 - v_3) + r_3 v_2) + r_3 r_6 + r_2 r_6 + (r_3 + r_2) r_4) v_1}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)}, i_2 = \frac{-(r_4(r_6 v_3 + r_1 v_3 + r_3 v_2) + r_1 r_6 v_3 + (r_3 r_6 + r_3 r_4) v_1)}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)}, i_3 = \frac{-(r_4(-r_6 v_3 - r_1 v_3 + r_2(v_2 - v_3)) - r_2 r_6 v_3 - r_1 r_6 v_3 + (r_2 r_6 + r_2 r_4) v_1)}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)}, i_4 = \frac{-(r_2(-r_6 v_3 - r_3 v_2 - r_1 v_2) - r_1 r_3 v_2 + (r_3 r_6 + r_2 r_6) v_1)}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)}, i_5 = 0, i_6 = \frac{-(r_4(-r_6 v_3 - r_1 v_3 + r_2(v_2 - v_3)) - r_2 r_6 v_3 - r_1 r_6 v_3 + (r_2 r_6 + r_2 r_4) v_1)}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)}, i_7 = \frac{(r_4(r_2(v_2 - v_3) + r_3 v_2) + r_2(r_3 v_2 + r_1 v_2) + r_1 r_3 v_2 + (r_3 + r_2) r_4 v_1)}{(r_4(r_2(r_6 + r_3 + r_1) + r_3 r_6 + r_1 r_3) + r_2(r_3 r_6 + r_1 r_6) + r_1 r_3 r_6)} \}$$

You can note that the single solution here for  $I_5$  is zero. This is because the wire (defm) shorts out  $R_5$ . The circuit is therefore can be simplified. In a simpler form, it looks like this:



We can also simplify the section (abc) so that the circuit looks like this:



Here, we need 3 loops and 2 junctions:

$$(ada): +I_3 \cdot R_3 - V_3 - I_2 \cdot R_2 = 0$$

$$(adha): I_2 \cdot R_2 + I_4 \cdot R_4 + V_1 - I_1 \cdot R_1 = 0$$

$$(hdgh): -I_4 \cdot R_4 - I_7 \cdot R_6 + V_2 = 0$$

$$@a: I_1 + I_2 + I_3 = 0$$

$$@d: I_4 - I_2 - I_7 - I_3 = 0$$

At Wims:

$$I_3 \cdot R_3 - V_3 - I_2 \cdot R_2 = 0$$

$$I_2 \cdot R_2 + I_4 \cdot R_4 + V_1 - I_1 \cdot R_1 = 0$$

$$-I_4 \cdot R_4 - I_7 \cdot R_6 + V_2 = 0$$

$$I_1 + I_2 + I_3 = 0$$

$$I_4 - I_2 - I_7 - I_3 = 0$$

Parameters:  $R_1, R_2, R_3, R_4, R_6, V_1, V_2, V_3$

If you solve these, the results are fairly nasty.

$$\left\{ \begin{aligned} i_1 &= -(r_2(r_6+r_4) v_3 + r_3((-r_6-r_4) v_1 - r_4 v_2) + r_2((-r_6-r_4) v_1 - r_4 v_2)) / \\ &(r_3(r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 r_6) + r_2(r_4(r_6+r_1) + r_1 r_6)), i_2 = -((r_4(r_6+r_1) + r_1 r_6) v_3 + r_3(r_4 \\ &v_2 + (r_6+r_4) v_1)) / (r_3(r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 r_6) + r_2(r_4(r_6+r_1) + r_1 r_6)), i_3 = \\ &((r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 r_6) v_3 + r_2((-r_6-r_4) v_1 - r_4 v_2)) / (r_3(r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 \\ &r_6) + r_2(r_4(r_6+r_1) + r_1 r_6)), i_4 = (r_2 r_6 v_3 + r_3(r_2 v_2 + r_1 v_2 - r_6 v_1) + r_2(r_1 v_2 - r_6 v_1)) / \\ &(r_3(r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 r_6) + r_2(r_4(r_6+r_1) + r_1 r_6)), i_7 = -(r_2 r_4 v_3 + r_3(-r_4 v_2 - r_2 v_2 - r_1 \\ &v_2 - r_4 v_1) + r_2(-r_4 v_2 - r_1 v_2 - r_4 v_1)) / (r_3(r_2(r_6+r_4) + r_4(r_6+r_1) + r_1 r_6) + r_2(r_4(r_6+r_1) + r_1 r_6)) \end{aligned} \right\}$$

What if, however, you now asked a question such as: supposing all resistors are the same, what value of  $V_1$  would cause  $I_1$  to become zero?

At Wims, enter:

$$I_3 * R - V_3 - I_2 * R = 0$$

$$I_2 * R + I_4 * R + V_1 - I_1 * R = 0$$

$$-I_4 * R - I_7 * R + V_2 = 0$$

$$I_1 + I_2 + I_3 = 0$$

$$I_4 - I_2 - I_7 - I_3 = 0$$

Parameters: R,  $V_1, V_2, V_3$

I'll assign a value of R to each of these resistors. The result is this:

$$\{ i_1 = -(v_3 - v_2 - 2v_1)/(4r), i_2 = -(3v_3 + v_2 + 2v_1)/(8r), i_3 = (5v_3 - v_2 - 2v_1)/(8r), i_4 = (v_3 + 3v_2 - 2v_1)/(8r), i_7 = -(v_3 - 5v_2 - 2v_1)/(8r) \}$$

Notice that 
$$I_1 = \frac{-(V_3 - V_2 - 2V_1)}{4R}$$

If we want  $I_1$  to be zero, then we solve:

$$V_3 - V_2 - 2V_1 = 0 \Rightarrow V_1 = \frac{V_3 - V_2}{2}$$

Notice for this particular case, we would have  $I_3 = -I_2$  as is required. Also note that  $I_4$  and  $I_7$  are also the same. Essentially we have broken the circuit into two branches. You can now find the power dissipated in each element of the circuit and the total power dissipated.

You can also find this out at Wims: Enter:

$$I_3 * R - V_3 - I_2 * R = 0$$

$$I_2 * R + I_4 * R + V_1 - I_1 * R = 0$$

$$-I_4 * R - I_7 * R + V_2 = 0$$

$$I_1 + I_2 + I_3 = 0$$

$$I_4 - I_2 - I_7 - I_3 = 0$$

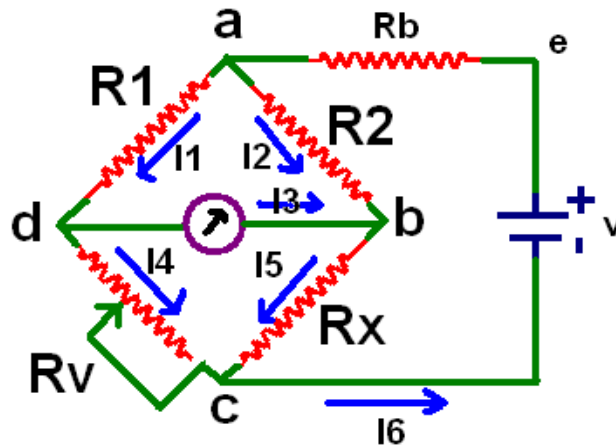
$$I_1 = 0$$

parameters: R,  $V_2, V_3$

Solution:  $\{ i_1 = 0, i_2 = -v_3/(2r), i_3 = v_3/(2r), i_4 = v_2/(2r), i_7 = v_2/(2r), v_1 = (v_3 - v_2)/2 \}$

### The Wheatstone Bridge Circuit:

Measurement of the unknown resistance  $R_x$ .



When the bridge is balanced,  $I_3$  is zero which means there is no potential difference between d and b.

$$(adcea): -I_1 R_1 - I_4 R_v + V - I_6 R_b = 0$$

$$(abcea): -I_2 R_2 - I_5 R_x + V - I_6 R_b = 0$$

$$(adba): -I_1 R_1 - I_3 R + I_2 R_2 = 0$$

$$@a: I_6 - I_1 - I_2 = 0$$

$$@c: I_4 + I_5 - I_6 = 0$$

If at Wims you enter:

$$-I_1 R_1 - I_4 S + V - I_6 B = 0$$

$$-I_2 R_2 - I_5 X + V - I_6 B = 0$$

$$-I_1 R_1 - I_3 R + I_2 R_2 = 0$$

$$I_6 - I_1 - I_2 = 0$$

$$I_4 + I_5 - I_6 = 0$$

Parameters:  $R, R_1, R_2, V, B, S, X$

You will find that the system has infinite solutions. However, if the bridge is balanced, then  $I_3 = 0$ . So that can be added to the system of equations, but note that if  $I_3 = 0$  then  $I_1 = I_4$  and  $I_2 = I_5$ . Add these additional two conditions to the solver gives for  $I_3$ :

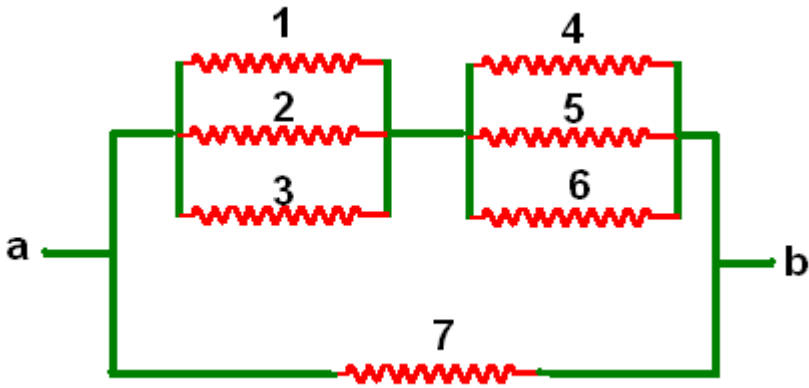
$$i_3 = v(r_2 s - r_1 x) / (s(r x + r_2 + b r) + r_1(r x + r_2 + b r) + b(r x + r_2))$$

Which means now that since  $I_3$  is zero, we must have:

$$\text{The unknown resistance: } R_2 R_v - R_1 R_x = 0 \Rightarrow R_x = \frac{R_2}{R_1} R_v .$$

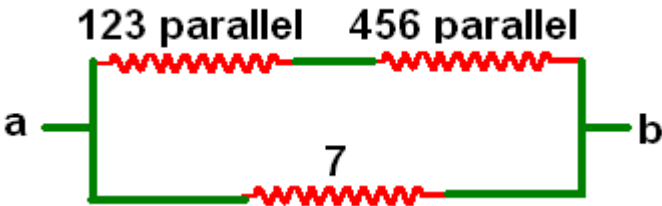
**Reduction of circuits**

It is possible with many simple circuits to reduce the circuits without Kirchoff's laws. Let me show you some examples of how to do this:

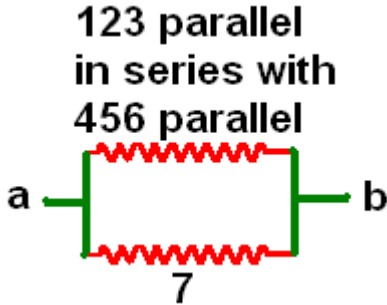


Reduce this circuit in a step-wise method recognizing series and parallel combinations of resistors:

Step 1:



Step 2:



The last step in finding the equivalent resistance is then clear. If you would like to see a numerical example of this, let the resistors be the value of the number that I've assigned to them. Then

$$\begin{aligned}
 123 \text{ parallel} &= \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} = \frac{1}{\frac{6}{6} + \frac{3}{6} + \frac{2}{6}} = \frac{1}{\frac{11}{6}} = \frac{6}{11} \\
 456 \text{ parallel} &= \frac{1}{\frac{1}{4} + \frac{1}{5} + \frac{1}{6}} = \frac{1}{\frac{3}{120} + \frac{24}{120} + \frac{20}{120}} = \frac{1}{\frac{74}{120}} = \frac{120}{74}
 \end{aligned}$$

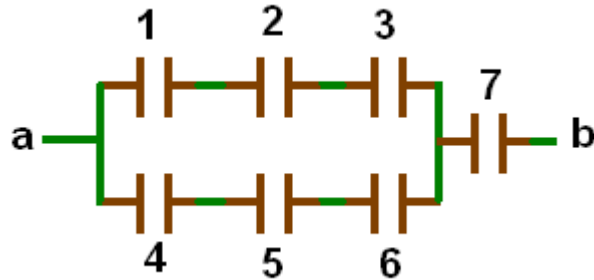
$$(123 \text{ parallel}) \text{ series } (456 \text{ parallel}) = \frac{6}{11} + \frac{120}{74} = \frac{444 + 1320}{814} = \frac{1764}{814}$$

The final step is to calculate the equivalent resistance of the last 2:

$$R_{\text{eq}} = \frac{1}{\frac{1764}{814} + \frac{1}{7}} = \frac{1}{\frac{12348 + 814}{5698}} = \frac{5698}{13162} = 0.43 \Omega$$

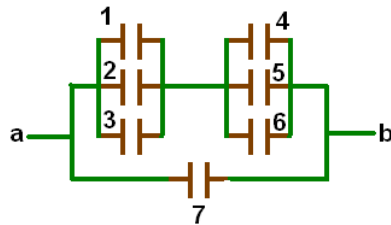
You can work backwards from here to find the current through each resistor. It is somewhat important to be able to recognize these series and parallel combinations of resistors.

The same technique also works for capacitors.  
 in fact, it the same only different .... what is the equivalent capacitance of this circuit?



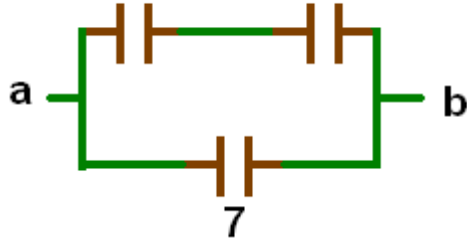
where each of the capacitances is given in  $\mu\text{f}$ ? (without doing further calculations)  
 The answer after you think about it for a while is clearly  $0.43 \mu\text{f}$ .

Let's do the same thing for a similar arrangement of capacitances:



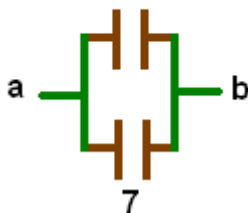
The first reduction is:

**123 parallel    456 parallel**



Then

**123 parallel  
 series  
 456 parallel**



You can calculate this equivalent capacitance:

$$(123 \text{ parallel}) = 1 + 2 + 3 = 6 \mu\text{f}$$

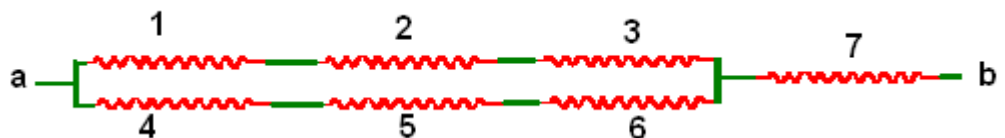
$$(456 \text{ parallel}) = 4 + 5 + 6 = 15 \mu\text{f}$$

$$(123 \text{ parallel}) \text{ series } (456 \text{ parallel}) = \frac{1}{\frac{1}{6} + \frac{1}{15}} = \frac{1}{\frac{15}{90} + \frac{6}{90}} = \frac{90}{21}$$

The equivalent capacitance is then given by:

$$\frac{90}{21} + 7 = 11.29 \mu\text{f}$$

Ok, what about this circuit



Without calculation, if the numbers represent Ohms, then the equivalent resistance is

$$11.29 \Omega$$

Unfortunately, these types of identifications (i.e. the connection between capacitive and resistive circuits in the analysis) is of limited utility which is why they're probably not often shown.