Bohr model worksheet and notes 2016

What I am going to show here is how to obtain the optical spectrum from such atoms.

Although this is largely classical, the results are astoundingly good.

The model: assume a model of an electron in orbit about a massive nucleus with charge +e. The electron has a charge -e

(1) Write down the force of attraction between the nucleus and the electron from Coulomb's law. This force will be a central force.

Coulomb's law says: $F=k\frac{q_1 q_2}{r^2}=k\frac{e^2}{r^2}$

k is Coulomb's constant and has the value **$9\text{x}10^9$ Nm^2/C^2**

e is the charge on the electron and has the value **$1.6\text{x}10^{-19}$ C**

Here I deal a bit with only uniform circular motion

(2) Write down the form of the centripetal force from Newton's law.

The electron orbiting the nucleus will respond to this central force by Newton's law:

$$F=m_2\frac{v^2}{r}$$

where v is the tangential speed of the orbiting electron.

The mass of the electron is **$m_e=9.11\text{x}10^{-31}$ kg** (Note: **$m_p=1.66\text{x}10^{-27}$ kg**)

(3) Multiply the top and the bottom of this expression by mr^2.

We can rewrite this in terms of something which is more like the moment of inertial:

$$F=m_e\frac{v^2}{r}=m_e\frac{m_e r^2}{m_e r^2}\frac{v^2}{r}=\frac{m_e^2 r^2 v^2}{m_e r^3}$$

(4) Identify L=rp=mvr and write (3) in terms of L

The angular momentum for a velocity which is perpendicular to a radius is given by:

$$L=rp=m_e v r \Rightarrow F=\frac{L^2}{m_e r^3}$$

(5) Equate (1) and (4) since these are the predominate forces acting in the atom.

The force producing the uniform circular motion is the Coulomb's force. That is why we equate these two results.

$$k\frac{e^2}{r^2}=\frac{L^2}{m_e r^3}$$

(6) Solve (5) for the radius of orbit in terms of L.

We want to place the angular momentum on one side by itself because we are going to apply quantization to this. Thus:

$$k\frac{e^2}{r^2}=\frac{L^2}{m_e r^3}\Rightarrow L^2=k m_e r^3\frac{e^2}{r^2}=k m_e r e^2$$

(7) Apply Bohr's quantization condition where L=nh/2π =nħ (ħ=0127) to find a new expression for r.

This was Bohr's assumption: the angular momentum appears only in discrete units. If one considers the electron to be composed of matter waves using DeBroglie's hypothesis, it turns out that only these particular values of angular momentum lead to closed orbits.

$$L^2=\left[\frac{nh}{2\pi}\right]^2=[n\hbar]^2=k m_e r e^2\Rightarrow r=\frac{[n\hbar]^2}{k m_e e^2};n=1,2,...$$

Also note:

$$ke^2=\frac{L^2}{m_e r}$$

h is known as **"Planck's Constant**" and has the value of **6.626068 × 10^{-34} m^2 kg / s and ħ is h/2 π.**

(8) Find the smallest radius and equate this to the constant a_0.Also, write ke^2 in terms of a_0.

The smallest radius will correspond to the case where n=1. This is, however, called a_0 which is also named the Bohr radius. This can be determined now:

$$A_0=\frac{[1\hbar]^2}{k m_e e^2}=\frac{h^2}{4\pi^2 k m_e e^2}$$

You can show that the result here is **a_0=0.529 x10^{-10} m. Note that an unusually nice combination of constants is the fine structure constant:**

$$\alpha=\frac{e^2}{\hbar c}\approx\frac{1}{137}$$

(9) Write r in terms of a_0 and write ke^2 in terms of L, m and a_0 using (6).
We can now use this to make the expression for the radius of any orbit appear much simpler. This is given by: $r=n^2 a_0$. Furthermore, we can now also write:

$$L^2 = k m_e e^2 [n a_0] \Rightarrow k e^2 = \frac{L^2}{m_e n a_0}$$

(10) Use Bohr's quantization condition to find v in terms of r and m and from (9) in terms of a_0, n and m.
Since the angular momentum is given by L=mvr, we are able to rewrite the expression as:

$$L = m_3 r v = n\hbar \Rightarrow m_e n^2 a_0 v = n\hbar \Rightarrow v = \frac{h}{2\pi m_e n a_0}$$

(11) Find the total K from $(1/2)mv^2$.
We can now find the kinetic energy of the electron in any Bohr orbital as:

$$K = \frac{1}{2} m_e v^2 = \frac{1}{2} m_e \left[\frac{h}{2\pi m_e n a_0}\right]^2 = \frac{h^2}{8\pi^2 m_e n^2 a_0^2}$$

(12) Find the total U at a particular r. Write U in terms of $(h/2\pi)$, m and a_0 using (9).

The potential is the coulomb potential. This is thus given by:

$$U = -k\frac{e^2}{r} = \frac{\left[\frac{L^2}{m_e n^2 a_0}\right]}{n^2 a_0} = \frac{-L^2}{m_e n^4 a_0^2} = -\frac{h^2}{4\pi^2 m_e n^2 a_0^2}$$

(13) Use your results from (11) and (12) to find the total energy of the electron in the nth orbital radius.

$$E_n = K + U = \frac{h^2}{4\pi^2 m_e n^2 a_0^2}\left[\frac{1}{2} - 1\right] = -\frac{h^2}{8\pi^2 m_e n^2 a_0^2}$$

(14) Provide a value for the ionization energy E_0 of the electron from (13). Then write E in terms of n and E_0.

The energy in (13) is negative since the electron is bound. In order to remove the electron, energy must be added to make the energy zero. This amount of energy is called the ionization energy and is equal to the energy required to move the electron from n=1 to infinity. Thus, the ionization energy is given by:

$$\Delta E = E_{n=\infty} - E_{n=1} = \frac{h^2}{8\pi m_e n^2 a_0^2} \equiv E_\infty$$

This value in “electron volts (eV)” is 13.6 eV. In Joules, this value is 2.2×10^{-18}J since
1 electron volt = 1.60217646 × 10^{-19} Joules

Notice that with this value, we can now write the energy levels of the Bohr atom in a very simple way:

$$E_n = \frac{-E_\infty}{n^2} \quad .$$

(15) According to Bohr, a photon will be emitted as the result of a transition between two energy levels. For the case of the final state being n=2, we will have the visible spectrum of the atom resulting (if the atom is hydrogen). Calculate the energy of transition between a level with n>2 to the level n=2.

$$\Delta E_{n\rightarrow 2}=E_{n=\infty}\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

(16) Using E=hf, write (14) in terms of inverse wavelength.

The energy of a photon was hypothesized to be E=hf by Planck and later Einstein also verified this form with the photoelectric effect. Here, f is the frequency of the photon. Since light travels at c, we thus have the energy of the photon as $E=\frac{hc}{\lambda}$. The energy of a photon given off by a transition like this would be:

$$\frac{hc}{\lambda}=E_{\infty}\left[\frac{1}{2^2}-\frac{1}{n^2}\right]\Rightarrow\frac{1}{\lambda}=\frac{E_{\infty}}{hc}\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

(17) Compute your result to the Balmer series and thus provide a value for the Rydberg constant R.

Early on, it was observed that a very good fit to the visible spectrum from hydrogen obeys the following relationship:

$$\frac{1}{\lambda}=R_{\infty}\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

A direct comparison with the Bohr model then gives the Rydberg constant:

$$R_{\infty}=\frac{E_{\infty}}{hc}$$

The Rydberg constant has the value: $10\,973\,731.6\ m^{-1}$.

(18) Determine an expression for the fine structure constant $\alpha=v_1/c$ (which is equal to 1/137). It is really remarkable that so many constants fit together to produce a simple result here. The speed of the electron in the first Bohr orbital can be easily obtained now:

$$L=m_e vr\Rightarrow v=\frac{L}{m_e r}\Rightarrow v_1=\frac{nh}{2\pi m_e n^2 a_0}=\frac{h}{2\pi m_e a_0}\Rightarrow\frac{v_1}{c}=\frac{h}{2\pi m_e a_0 c}\equiv\alpha$$

It is not too hard to show $\alpha=\frac{e^2}{\hbar c}$ and both results provide a remarkable combination of constants into one easily remembered value (1/137).