

Physics 250: Unquiz 03

A parallel plate capacitor has one plate of area A located at $z=0$ and the other plate of area A located at $z=d$. There is a surface charge $+\sigma$ on the plate at $z=0$ and $-\sigma$ on the plate at $z=d$.

(a) Make a sketch showing $\vec{E}$ lines.

(b) Solve for the Electric field between the plates in terms of σ and ϵ_0 .

(c) Calculate the potential difference between the plates; noting that since $\vec{E}$ is uniform between the plates, $-\vec{\nabla} V = \vec{E} \Rightarrow -\frac{\Delta V}{\Delta z} \hat{z} = \vec{E}$.

(d) Calculate the total amount of charge separated, Q in terms of σ and A .

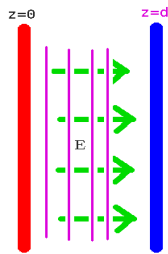
(d) The capacitance is defined by $C = \frac{Q}{V}$. Calculate the capacitance of the parallel plate capacitor in terms of ϵ_0 , A and d .

(e) When a capacitor is charged from 0 potential to a maximum potential of V , find the work done, and thus the stored energy by $W = \int_{q=0}^Q \frac{q}{C} dq \equiv U$. Express this also in terms of Q and V .

(f) Write the result in (e) in 2 other ways, one involving Q , and the other involving E and the interior volume of the capacitor.

(g) From your expression for the total stored energy (U) obtained in (f), find an expression for the energy density inside the parallel plate capacitor in terms of ϵ_0 and E .

A parallel plate capacitor has one plate of area A located at $z=0$ and the other plate of area A located at $z=d$. There is a surface charge $+\sigma$ on the plate at $z=0$ and $-\sigma$ on the plate at $z=d$.



(a) Make a sketch showing $\vec{E}$ lines.

In this diagram, the electric field lines are “green” and also shown are equipotential surfaces at right angles to the field lines.

(b) Solve for the Electric field between the plates in terms of σ and ϵ_0 .

Choose a Gaussian cylinder with ends of area A' oriented with the ends parallel to the planes. One end is between the capacitor and the other is outside. The only flux is through one end. The charge enclosed is $\sigma A'$. Then:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} \Rightarrow EA' = \frac{\sigma A'}{\epsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0} \hat{x}$$

(c) Calculate the potential difference between the plates; noting that since $\vec{E}$ is uniform between the plates, $-\vec{\nabla} V = \vec{E} \Rightarrow -\frac{\Delta V}{\Delta z} \hat{z} = \vec{E}$.

$$-\frac{\Delta V}{\Delta z} \hat{z} = \vec{E} \quad . \quad -\Delta V \hat{z} = \vec{E} \Delta z = E(0-d) \hat{z} \Rightarrow \Delta V = Ed$$

(d) Calculate the total amount of charge separated, Q in terms of σ and A .
 $Q = \sigma A$

(d) The capacitance is defined by $C = \frac{Q}{V}$. Calculate the capacitance of the parallel plate capacitor in terms of ϵ_0 , A and d .

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{\sigma A}{\frac{\sigma}{\epsilon_0} d} = \epsilon_0 \frac{A}{d}$$

(e) When a capacitor is charged from 0 potential to a maximum potential of V , what is the average potential that is established? Multiply this by the total charge separated to obtain the work required to charge a capacitor in terms of Q and V and in terms of Q and C .

$$\langle V \rangle = \frac{1}{2} V \Rightarrow W = \frac{1}{2} Q V = \frac{Q^2}{2C} \equiv U$$

(f) Write the result in (e) in 2 other ways, one involving V , and the other involving E and the interior volume of the capacitor.

$$U = \frac{1}{2} (CV) V = \frac{1}{2} C V^2 : U = \frac{1}{2} \epsilon_0 \frac{A}{d} (Ed)^2 = \frac{1}{2} \epsilon_0 E^2 (Ad) = \frac{1}{2} \epsilon_0 E^2 (\text{volume})$$

(g) From your expression for the total stored energy (U) obtained in (f), find an expression for the energy density inside the parallel plate capacitor in terms of ϵ_0 and E .

$$u \equiv \frac{U}{\text{volume}} = \frac{1}{2} \epsilon_0 E^2$$