7.1.2 emf

According to the discussion from your author (see pages 791, 792), when a battery is connected to the circuit, the battery exerts a force on the charges (and it is perhaps local to the battery). This force is, by means of the electrostatic field, translated around the (closed) circuit. Kinks (or, let's say a cut) in the circuit would result in an accumulation of charge and this an electrostatic field that would eventually equal this force and current would not flow.

This then means that the force on the charges is given by:

$$f = f_s + E$$

where f_s represents the force from the source (a battery).

If the system is in equilibrium, the net force on the charges is zero so:

$$f_s = -E$$

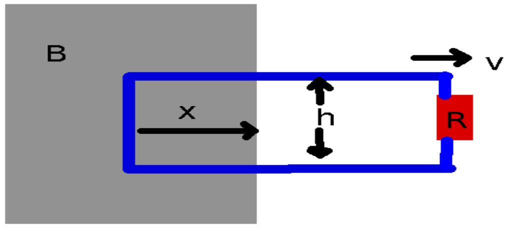
We can find the potential difference between the terminal of the battery as:

$$V = -\int_a^b \vec{E} \cdot d\vec{L} = \int_a^b \vec{f}_s \cdot d\vec{L} \equiv \text{emf}$$

Your author prefers to define emf in this way and not as work per unit charge.

7.1.3 motional emf

Consider the following circuit (figure 7.10 in the text):



The Lorentz force from the magnetic field is acting on the charges in the wire:

$$\vec{F}=q\vec{v}\times\vec{B}$$

If the field is pointing out of the screen, then the charges in the segment of length h in the field will experience a magnetic force along the length of the wire. The direction can be predicted for positive or negative charges by the right hand rule. For positive charges, the direction would be downward and there would thus be a counter clockwise direction around the loop of the conventional current. Reality check: according to Physics 250, Lenz's law would say that the direction of the induced current would be in such a direction so that the magnetic field is pointing out of the board. The direction of current would then be, in agreement, counterclockwise.

Here, the size of the magnetic force is then $F=qvB \Rightarrow E=vB$.

The potential difference across the resistor is then: $emf=\int E\,dl=vbh$.

We can write this in terms of the magnetic flux through the circuit:

$$\Phi_M=\oint\!\!\oint \vec{B}\cdot d\vec{A}=Bh(\text{length})$$

Then we find that: $\frac{d\Phi_M}{dt}=Bh\frac{d\,\text{length}}{dt}=-Bvh$

(negative because the length of the circuit in the magnetic field is decreasing).

... as the x coordinate increases, the length in the magnetic field decreases: you can show this with a substitution of variables.

7.2.2 This gives us Faraday's law:

$$emf=\frac{-d\Phi_M}{dt}\ .$$

However, if the circuit were fixed and the magnetic field were changing, in this situation the emf can not be obtained simply by looking at motional emf since the charges are not moving. This means that, in fact, we have not derived Faraday's law in all circumstances. It was an inspiration of Faraday that has been verified by experiment. We can express this in differential form:

$$emf=\int \vec{E}\cdot d\vec{L}=\frac{-d\Phi_m}{dt}=-\oint\!\!\oint\frac{d\vec{B}}{dt}\cdot d\vec{A}:\Rightarrow\oint\!\!\oint(\vec{\nabla}\times\vec{E})\cdot d\vec{A}=-\oint\!\!\oint\left(\frac{d\vec{B}}{dt}\right)\cdot d\vec{A}\Rightarrow\vec{\nabla}\times\vec{E}=\frac{-\partial\vec{B}}{\partial t}$$

It is Lenz's law (nature abhors a change in magnetic flux) that allows us to easily keep track of the direction of the induced current and electric fields.