

Physics 250: Unquiz #2

A sphere of uniform charge density ρ and total charge Q has a radius a . At the center of the sphere is a solid metal sphere of radius b , $b < a$. Find the electric field vector in the three regions.

For $r > a$, choose a Gaussian sphere of radius r , $r > a$. On the Gaussian sphere, the electric field is parallel to the normal to the area and uniform. The total charge enclosed is Q . By Gauss's law:

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} : \Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = E(4\pi r^2) : Q_{\text{enc}} = Q \Rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\vec{E}_{\text{outside}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

For $r > b$, $r < a$, choose a Gaussian sphere of radius r , $r > b$, $r < a$. On the Gaussian sphere, the electric field is parallel to the normal to the area and uniform.

$$Q_{\text{enc}} = \rho \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3 \right) = \frac{4}{3}\pi \rho (r^3 - b^3) : \Phi_E = E(4\pi r^2) \Rightarrow E = \frac{\rho}{3\epsilon_0} \left(r - \frac{b}{r^2} \right)$$

At a radius r ,

$$\vec{E}_{r > b, r < a} = \frac{\rho}{3\epsilon_0} \left(r - \frac{b}{r^2} \right) \hat{r}$$

For $r < b$, no net charge is enclosed. In general the area here is not zero so the electric field for $r < b = 0$. A solid metal sphere can not have a net electric field in its interior.