	Physics 220: Worksheet 3A	Name: ________________

Supplementary problems for electrostatics (Improved Spring 2015)

1. Three identical charges each with $q=1\mu C$ are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the center of the triangle.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge $Q=1\mu C$ on one plate and $-Q=-1\mu C$ on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

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1. Three identical charges each with $q=1\mu C$ are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the center of the triangle.

Improved for Spring 2015

The hard part here is to locate coordinates of the points. Let the triangle have a side of length L. Locate one point at the origin. Locate the second point at (L,0). We need to find the coordinates of the center, and the 3rd point. The x coordinate of the 3rd point is x=L/2. The y coordinate is then given by:

$$L^2=\left(\frac{L}{2}\right)^2+y^2 \Rightarrow y^2=L^2-\frac{L^2}{4}=3\frac{L^2}{4}\Rightarrow y=\frac{\sqrt{3}}{2}L$$

.

The coordinates of the center of the triangle are then: $x=\left(\frac{L}{2,}\frac{\sqrt{3}}{4}L\right)$. In our present case, L=1 m. We thus have the coordinates for this particular orientation:

$$1:(1\mu;0,0),2:(1\mu,1,0),3:(1:.5,\frac{\sqrt{3}}{2}),p:(0.5,\frac{\sqrt{3}}{4})$$

Calculate the following vectors: $\vec{r}_{1,}\vec{r}_{2,}\vec{r}_{3,}\vec{r}_p,\vec{r}_{1p},\vec{r}_{2p},\vec{r}_{3p}$

These vectors are:

$$\vec{r}_p=0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_1=0\,\hat{x}+0\,\hat{y}:\vec{r}_{1p}=\vec{r}_p-\vec{r}_1=-0.5\,\hat{x}-\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_2=1\,\hat{x}+0\,\hat{y}:\vec{r}_{2p}=\vec{r}_p-\vec{r}_i=\left(0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-(1\,\hat{x}+0\,\hat{y})=-0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}$$

$$\vec{r}_3=0.5\,\hat{x}+\frac{\sqrt{3}}{2}\hat{y}:\vec{r}_{3p}=\vec{r}_p-\vec{r}_i=\left(0.5\,\hat{x}+\frac{\sqrt{3}}{4}\hat{y}\right)-\left(0.5\,\hat{x}+\frac{\sqrt{3}}{2}\hat{y}\right)=0\,\hat{x}-\frac{\sqrt{3}}{4}\hat{y}$$

Now write E directly.

$$\vec{E}_p=k\,\mu\left[\hat{x}(-0.5-0.5+0)+\hat{y}\left(\frac{-\sqrt{3}}{4}+\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}\right)\right]=k\,\mu\left[-1\,\hat{x}-\frac{\sqrt{3}}{4}\hat{y}\right]$$

$$\Rightarrow\vec{E}_p=[-8990\,\hat{x}-3892\,\hat{y}]\frac{N}{C}$$

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2. A square has charges at the following locations and values:

(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).

Find the value of the vector electric field at the origin.

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Notice that if $\vec{r}_p = 0\hat{x} + 0\hat{y}$, then $\vec{r}_{ip} = \vec{r}_p - \vec{r}_i = \vec{0} - \vec{r}_i = -\vec{r}_i$. We can use this to write down the electric field directly:

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+1\hat{y})}{[1^2+1^2]^{3/2}} + \frac{2(-1\hat{x}+1\hat{y})}{[2]^{3/2}} + \frac{3(-1\hat{x}-1\hat{y})}{2\sqrt{2}} + \frac{4(\hat{x}-\hat{y})}{2\sqrt{2}}\right]$$

$$\Rightarrow \vec{E}_p = \frac{k\mu}{2\sqrt{2}}\left[\hat{x}(1-2-3+4) + \hat{y}(1+2-3-4)\right] = \frac{k\mu}{2\sqrt{2}}(-4\hat{y}) = \frac{-2k\mu}{\sqrt{2}}\hat{y} = -\sqrt{2}k\mu\hat{y} = -1213\hat{y}\frac{N}{C}$$

3. Three charges are arranged as follows:

(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).

Find the vector electric field at the origin.

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This satisfies the condition $\vec{r}_p = \vec{0}$. We can write the field directly.

$$\vec{E}_p = k\mu\left[\frac{1(1\hat{x}+0\hat{y})}{\left[1^2+0^2\right]^{3/2}} + \frac{-1(-1\hat{x}+0\hat{y})}{1} + 1(0\hat{x}-1\hat{y})\right] = k\mu\left[\hat{x}(1+1+0) + \hat{y}(0+0-1)\right] = k\mu\left[2\hat{x} - 1\hat{y}\right]$$

$$\Rightarrow \vec{E}_p = \left[17980\hat{x} - 8990\hat{y}\right]\frac{N}{C}$$

4. A sphere of radius a has a charge Q spread uniformly over its volume and at the center of the sphere is a charge –Q. Find the vector electric field inside and outside the sphere.
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Outside both spheres, the electric field is zero because the net charge enclosed is zero.
Choose a Gaussian surface on the shape of a sphere centered on the distribution. On this Gaussian surface: $\vec{A}=A\hat{r}:|\vec{E}|=\text{constant}:\vec{E}=|\vec{E}|\hat{r}$. Ignore for now the charge at the center. From Gauss's law we have: $\Phi_E=\frac{Q_{enc}}{\epsilon_0}$. Outside, $Qenc=Q$ and

$$\Phi_E=\sum_{\Delta A_i}\vec{E}_i\cdot\Delta\vec{A}_i=\sum_{\Delta A_i}E_i\,\Delta A_i=E\sum_{\Delta A_i}\Delta A_i=E\left(4\pi r^2\right)\Rightarrow E\left(4\pi r^2\right)=\frac{Q}{\epsilon_0}\Rightarrow\vec{E}_{out}=\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

Inside the sphere, we then have: $\Phi_E=E\left(4\pi r^2\right)$ again, but $Q_{enc}=\frac{Q}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi r^3\right)=Q\frac{r^3}{a^3}$. We find the electric field inside the sphere to be then: $E\left(4\pi r^2\right)=\frac{Q}{\epsilon_0}\frac{r^3}{a^3}\Rightarrow\vec{E}_{in}=\frac{Qr}{4\pi\epsilon_0 a^3}\hat{r}$.
Now it is necessary to superimpose the electric field from the charge at the center on each of the solutions. Thus:

$$\text{outside}:\vec{E}_{out}=\vec{0}:\text{Inside}:\vec{E}_{in}=\frac{Q}{4\pi\epsilon_0}\left[\frac{r}{a^3}-\frac{1}{r^2}\right]\hat{r}.$$

Notice that we could have observed the outside solution easier by noticing that no net charge is enclosed.

Inside the sphere: $Q_{enc}=4\pi\int_{r=0}^{r=r}\rho_0 r^2\,dr=4\pi\int_{r=0}^{r=r}\frac{\rho_0}{a}r^3\,dr=\pi\frac{\rho_0}{a}r^4$

So from Gauss's law: $E_{in}\left(4\pi r^2\right)=\frac{\pi}{\epsilon_0}\frac{\rho_0}{a}r^4\Rightarrow\vec{E}_{in}=\frac{\rho_0 r^2}{4\epsilon_0 a}\hat{r}=\frac{Qr^2}{4\pi\epsilon_0 a^4}\hat{r}$

Notice at the surface, the inside solution is equal to the outside solution.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?
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The surface charge density is given by $\sigma=\frac{Q}{A}=1\frac{\mu C}{m^2}$
Outside the capacitor note that the electric field is zero.
Choosing a cylindrical Gaussian surface of area A’ we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$EA'=\frac{\sigma A'}{\epsilon_0}\Rightarrow E=\frac{\sigma}{\epsilon_0}=\frac{1x10^{-6}}{8.85\,x\,10^{-12}}=1.13\,x\,10^5\frac{N}{C}.$$