Potential due to a finite wire (length +-L) along symmetry axis bisector

$$dq_i = \begin{Bmatrix} \lambda dx_i : |x_i| < L \\ 0 : |x_i| > L \end{Bmatrix}$$

$$V(\vec{r}_p) = \int_{\substack{all\\q}} k \frac{dq_i}{|\vec{r}_{ip}|} = k \int_{x=-L}^{x=L} \frac{\lambda dx_i}{\sqrt{x_i^2 + y_p^2}} = k\lambda \ln\left[x_i + \sqrt{x_i^2 + y_p^2}\right]\Bigg|_{-L}^{+L} = k\lambda \left\{ \ln\left[\frac{\sqrt{L^2 + y_p^2} + L}{\sqrt{L^2 + y_p^2} - L}\right] \right\}$$

Suppose you are very far from the wire: this should look like a point charge. Let's verify it:

$$\sqrt{L^2 + y_p^2} = y_p \sqrt{1 + \left(\frac{L}{y_p}\right)^2} \approx y_p \left\{ 1 - \frac{1}{2}\left(\frac{L}{y_p}\right)^2 \right\}$$

So:

$$\frac{y_p + L}{y_p - L} = \frac{(y_p + L)}{y_p}\left[\frac{1}{1 - \frac{L}{y_p}}\right] \approx \frac{(y_p + L)}{y_p}\left(1 + \frac{L}{y_p}\right) = \left(1 + \frac{L}{y_p}\right)^2$$

$$\ln\left(1 + \frac{L}{y_p}\right)^2 = 2\ln\left(1 + \frac{L}{y_p}\right) \approx 2\left[\frac{L}{y_p} - \frac{1}{2}\left(\frac{L}{y_p}\right)^2 + \ldots\right] \approx 2\frac{L}{y_p}$$

The potential is then:

$$V(y_p) \approx \frac{2k\lambda L}{y_p} = \frac{kQ}{y_p}$$

Of course, I've implicitly restricted y_p to lie in the positive region. More generally the potential would be:

$$V(y_p) \approx \frac{kQ}{|y_p|}$$