

Calculate the potential due to a charged ring of radius a along the symmetry axis.

$$V(\vec{r}_p) = \int_{\text{all } q} \frac{k dq_i}{|\vec{r}_p|}$$

coordinates:

$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} : \vec{r}_p = z_p \hat{z} : \vec{r}_p = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z} : |\vec{r}_p| = \sqrt{x_i^2 + y_i^2 + z_p^2} = \sqrt{a^2 + z_p^2}$$

dq :

$$dq_i = \lambda a d\phi$$

Then:

$$\vec{V}(\vec{r}_p) = \int_{\phi=0}^{\phi=2\pi} \frac{k\lambda a d\phi}{\sqrt{a^2 + z_p^2}} = \frac{2\pi k\lambda a}{\sqrt{a^2 + z_p^2}}$$