

Calculate the potential due to a charged disk of radius a along the symmetry axis.

$$V(\vec{r}_p) = \int_{\substack{\text{all} \\ q}} \frac{k dq_i}{|\vec{r}_p|}$$

coordinates:

$$\vec{r}_i = x_i \hat{x} + y_i \hat{y} : \vec{r}_p = z_p \hat{z} : \vec{r}_p = -x_i \hat{x} - y_i \hat{y} + z_p \hat{z} : |\vec{r}_p| = \sqrt{x_i^2 + y_i^2 + z_p^2} = \sqrt{s_i^2 + z_p^2}$$

dq :

$$dq_i = \sigma ds d\phi$$

Then:

$$V(\vec{r}_p) = \int_{s=0}^{s=a} \int_{\phi=0}^{\phi=2\pi} \frac{k \sigma ds d\phi}{\sqrt{s^2 + z_p^2}} = \int_{s=0}^a \frac{2\pi \sigma k}{\sqrt{s^2 + z_p^2}} s ds = 2\pi k \sigma \sqrt{s^2 + z_p^2} \Big|_0^a = 2\pi k \sigma \left\{ \sqrt{a^2 + z_p^2} - z_p \right\}$$

In the limit of large z , this should look like a point charge potential:

$$\sqrt{a^2 + z_p^2} = z_p \sqrt{1 + \left(\frac{a}{z_p}\right)^2} \approx z_p \left\{ 1 + \frac{1}{2} \left(\frac{a}{z_p}\right)^2 \right\} \Rightarrow V(\vec{r}_p) \approx 2\pi k \sigma \left\{ \frac{a^2}{2z_p} \right\} \approx k \frac{\pi a^2}{z_p} = k \frac{Q}{z_p}$$