Problems: 2.41

$$E = k\int_{x=-a}^{x=+a}\int_{y=-a}^{y=+a}\frac{\sigma\left[-x_i\hat{x}-y_i\hat{y}+z_p\hat{z}\right]dxdy}{\left[x_i^2+y_i^2+z_p^2\right]^{3/2}} = kz_p\hat{z}\int_{x=-a}^{x=+a}\int_{y=-a}^{y=+a}\frac{dxdy}{\left[x_i^2+y_i^2+z_p^2\right]^{3/2}}$$

Problem 2.42

$$\vec{E} = \frac{A\hat{r}+B\sin\theta\cos\varphi\hat{\varphi}}{r}\quad \vec{\nabla}\bullet\vec{E} = \frac{\rho}{\varepsilon_0}\quad \vec{\nabla}\bullet\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2V_r\right)+\frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)V_\theta\right)+\frac{1}{r\sin(\theta)}\frac{\partial V_\varphi}{\partial\varphi}$$

Problem 2.46

$V\left(\vec{r}_p\right) = A\frac{e^{-\lambda r}}{r}$. Find E, ρ, Q.

$$\vec{E} = -\vec{\nabla}V : \vec{\nabla}\bullet\vec{E} = \frac{\rho}{\varepsilon_0} : Q = 4\pi\int_{r=0}^{\infty}\rho\, r^2dr$$

$$\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial T}{\partial\theta}\hat{\theta}+\frac{1}{r\sin(\theta)}\frac{\partial T}{\partial\varphi}\hat{\varphi}$$

Problem 2.48: obtain the Child-Langmuir law (delayed)

Problem 2.45

$\rho(r) = Cr$: find U (2 ways) : Radius $= a$

$$E_{in} = \frac{4\pi\int_0^r Cr\,r^2dr}{\varepsilon_0 4\pi r^2} = \frac{4\pi C\frac{r^4}{4}}{\varepsilon_0 4\pi r^2} = C\frac{r^2}{\varepsilon_0} : E_{out} = \frac{4\pi C\frac{a^4}{4}}{\varepsilon_0 4\pi r^2} = C\frac{a^4}{\varepsilon_0 r^2}$$

$$U = \frac{4\pi}{2}\varepsilon_0\int_{r=0}^{a}C^2\frac{r^4}{\varepsilon_0^2}r^2dr+\frac{4\pi}{2}\varepsilon_0\int_{r=a}^{r=\infty}C^2\frac{a^8}{\varepsilon_0^2 r^4}r^2dr$$

$$Cap = \frac{Q}{V} = \frac{4\pi\int_{r=0}^{a}Cr\,r^2dr}{-\int_{\infty}^{a}C\frac{a^4}{\varepsilon_0 r^2}dr-\int_{a}^{0}C\frac{r^2}{\varepsilon_0}dr} : U = \frac{Q^2}{2C}$$