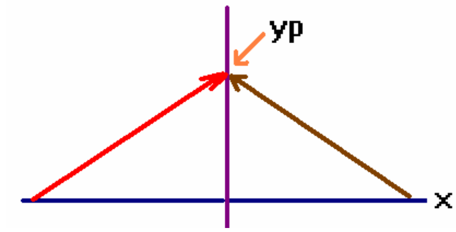
a line of charge of length 2L is aligned symmetrically along the x-axis and has a total charge Q. Find the electric field at points above the wire's central bisector.



I am going to define a charge density λ so that $Q = (2L)\lambda$.

The electric field from a small charge q_i located at x_i along the +x axis is given by:

$$\vec{E}_{p_{+x}} = k\frac{q_i}{x_i^2+y_p^2}\frac{-x_i\hat{x}+y_p\hat{y}}{\sqrt{x_i^2+y_p^2}}$$

The electric field from a small charge q_i located at x_i along the -x axis is given by:

$$\vec{E}_{p_{-x}} = k\frac{q_i}{x_i^2+y_p^2}\frac{x_i\hat{x}+y_p\hat{y}}{\sqrt{x_i^2+y_p^2}}$$

Let's choose to pick these charges two at a time (one from +x and one from –x) and add the electric fields together to get:

$$\vec{E}_p = k\frac{2q_i y_p}{\left[x_i^2+y_p^2\right]^{3/2}}\hat{y}$$

For the non-calculus students, this is as far as can be derived. The calculus students can obtain the final result:

$$\vec{E} = \int_{\substack{\text{all}\\\text{charges}}} d\vec{E} = 2k\int_{x_i=0}^{x_i=L}\frac{y_p dq}{\left[x_i^2+y_p^2\right]^{3/2}}\hat{y} = 2k\lambda y_p\int_{x_i=0}^{x_i-L}\frac{dx}{\left[x_i^2+y_p^2\right]^{3/2}}\hat{y}$$

This is pretty easy to evaluate if you use the integrator. The required input is:

(x^2+y^2)^(-3/2)

And the output is:

$$\left.\frac{x}{y_p^2\sqrt{x^2+y_p^2}}\right|_0^L = \frac{L}{y_p^2\sqrt{L^2+y_p^2}}$$

The electric field is then given by:

$$\vec{E} = 2k\lambda y_p\left\{\frac{L}{y_p^2\sqrt{L^2+y_p^2}}\right\}\hat{y}$$

As you get very far from the wire, you would expect this to look like a point charge. Let's confirm that.

$$\vec{E} = 2k\lambda y_p\left\{\frac{L}{y_p^2\sqrt{L^2+y_p^2}}\right\}\hat{y} = \frac{2k\lambda}{|y_p|}\frac{L}{y_p\sqrt{1+\left(\frac{L}{y_p}\right)^2}}\hat{y} \approx \frac{2k\lambda}{y_p^2}L\hat{y} = \frac{k(2\lambda L)}{y_p^2}\hat{y} = \frac{kQ}{y_p^2}\hat{y}$$

This is exactly the type of behavior that you would expect to see from a point charge.