

Electrostatic field due to charged thick disk

Charged disk: (height h, radius a, charge density ρ) along symmetry axis.

Charge density: $dq_i = \rho s ds d\phi dz_i$

$$\vec{r}_i = s \cos \phi \hat{x} + s \sin \phi \hat{y} + z_i \hat{z} : \vec{r}_p = z_p \hat{z} : \vec{r}_{ip} = -s \cos \phi \hat{x} - s \sin \phi \hat{y} + (z_p - z_i) \hat{z}$$

$$\vec{E} = \int_{s=0}^{s=a} \int_{\phi=0}^{\phi=2\pi} \int_{z_i=0}^{z_i=h} k\rho \frac{-s \cos \phi \hat{x} - s \sin \phi \hat{y} + (z_p - z_i) \hat{z}}{\left[s^2 + (z_p - z_i)^2 \right]^{\frac{3}{2}}} s ds d\phi dz_i = 2\pi k\rho \int_{s=0}^{s=a} \int_{z_i=0}^{z_i=h} \frac{(z_p - z_i) \hat{z}}{\left[s^2 + (z_p - z_i)^2 \right]^{\frac{3}{2}}} s ds dz_i$$

$$\vec{E} = 2\pi k\rho \left[\int_{z_i=0}^{z_i=h} \frac{(z_p - z_i) \hat{z}}{\sqrt{s^2 + (z_p - z_i)^2}} dz \right]_{s=0}^{s=a} = 2\pi k\rho \left[\int_{z_i=0}^{z_i=h} \frac{(z_p - z_i) \hat{z}}{\sqrt{a^2 + (z_p - z_i)^2}} dz - \int_{z_i=0}^{z_i=h} \frac{(z_p - z_i) \hat{z}}{\sqrt{(z_p - z_i)^2}} dz \right]$$

$$\vec{E} = 2\pi k\rho \hat{z} \left[-\sqrt{a^2 + (z_p - z_i)^2} \right]_{z_i=0}^{z_i=h} - \left[-\sqrt{(z_p - z_i)^2} \right]_{z_i=0}^{z_i=h}$$

$$\vec{E} = 2\pi k\rho \hat{z} \left[-\sqrt{a^2 + (z_p - h)^2} + \sqrt{a^2 + z_p^2} - \left[-\sqrt{(z_p - h)^2} + \sqrt{z_p^2} \right] \right]$$

$$\vec{E} = 2\pi k\rho \hat{z} \left[-\sqrt{a^2 + (z_p - h)^2} + \sqrt{a^2 + z_p^2} + \sqrt{(z_p - h)^2} - \sqrt{z_p^2} \right]$$

This can have many signs and values depending upon the relative values of z and h and also upon the absolute value of z.