

Lab 01A: Coulomb's law: Spring 2013

Charging with **electrophorus**

You will find additional information about this at the Exploratorium:

http://isaac.exploratorium.edu/~pauld/summer_institute/summer_day14electrostatic/Electrophorus.html

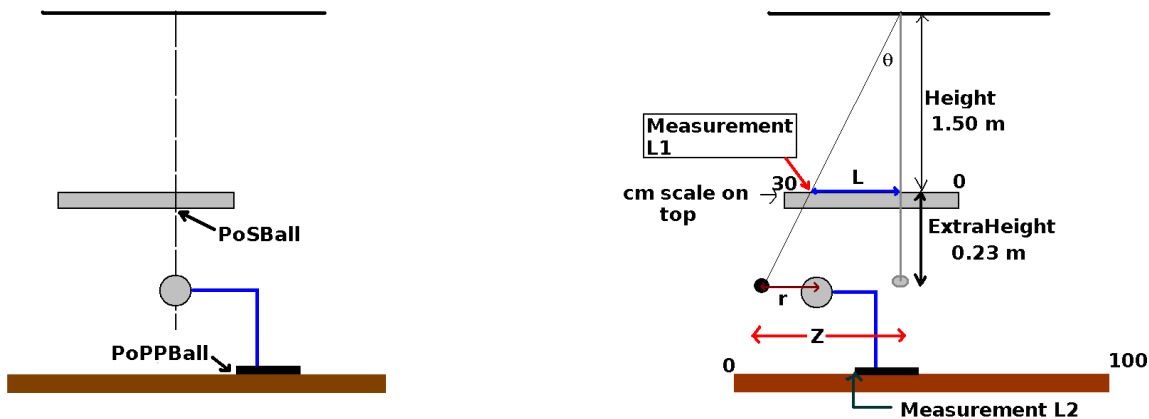
A pie pan and styrofoam can be used to charge effectively even in conditions of higher humidity. The idea is this: rub wool onto the styrofoam for about 1 minute (you do not need to be rough with it). Place the pie pan onto the top of the plate. Touch the pie pan (a spark will jump). Then remove your finger and separate the pie pan from the plate. We will use these again in lab01-B. Note that I have found a 1 inch block of styrofoam really works much better than styrofoam plates. We will, next week, confirm the sign of the charge. For now, just accept that it is either positively or negatively charged.

Experimental verification of the inverse square law

Without vectors, **Coulomb's law** states that the force of attraction or repulsion between charges is given by

$$F = \frac{kq_1q_2}{r^2}$$

where k is Coulomb's constant ($k=8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$, q_1 is the charge 1, q_2 is charge 2 and r is the distance of separation between the two charges). Incidentally, Coulomb's constant is often written $k=1/(4\epsilon_0)$ where ϵ_0 is the permittivity of free space and has the value $8.854187817 \times 10^{-12} \text{ C}^2/(\text{Nm}^2)$. We can know the value of ϵ_0 to so many decimal places by measurements involving the speed of light.



These two figures show the various defined quantities. As I have tried to set things up, the initial variables should be: **Height=1.50 m**, **ExtraHeight=0.23 m**, **PoSBall=0.23 m**, and **PoPPBall=0.45 m**. You will need to check most of these but I think you should trust my measurement of 1.50 m. Make sure after you finish your lab that the initial settings are returned. You will be making 10 measurements here of L1 and L2. I have chosen the two 0.23 settings for my own simplicity here.

Measurement	L1 [cm]	L2 [cm]
1		35
2		36
3		37
4		38
5		39
6		40
7		41
8		42
9		43
10		44

The analysis of this setup is as follows: the angle θ is measured from Height+Width and L:

$$\tan(\theta) = \frac{L}{\text{Height}}.$$

The magnitude of the restoring force when the small ball is displaced by an angle θ is given by:

$$F = mg\sin(\theta)$$

L is obtained from your measurement of L1 by:

$$L = |L1 - \text{PoSBall}|.$$

This, however, needs to be now extended so that it represents the measurement Z in the diagram.

$$\tan(\theta) = \frac{Z}{[\text{Height} + \text{ExtraHeight}]} \Rightarrow Z = [\text{Height} + \text{ExtraHeight}]\tan(\theta)$$

So we can find Z directly by:

$$Z = L \frac{[\text{Height} + \text{ExtraHeight}]}{\text{Height}} = L \left[1 + \frac{\text{ExtraHeight}}{\text{Height}} \right]$$

The separation between the two balls is given by:

$$r = |Z - |L2 - \text{PoPPBall}||$$

This is true because when L2 is PoPPBall (which is 0.45), Z=L. As L2 changes, this difference (L2-PoPPBall) measures then the exact amount that needs to be subtracted from Z to give the separation. If Coulomb's law is valid then we would have the following:

$$mg[\text{Height} + \text{ExtraHeight}]\sin(\theta) = \frac{kq_1q_2}{r^2}$$

In the analysis we thus have:

$$\sin(\theta) = \frac{kq_1q_2}{mg[\text{Height} + \text{ExtraHeight}]} \left[\frac{1}{r^2} \right] = \text{Constant} \left[\frac{1}{r^{\text{Power}}} \right]$$

What I want to do is to actually pull out that power dependence in the simplest way possible. For our purposes today, although we will determine the constant, it will not be essential (and it will vary from one experiment to another since it depends upon charge and mass). It is the power that we want to verify here.

To do this, I take the natural log of this equation:

$$\ln(\sin(\theta)) = \ln(\text{constant}) - (\text{Power}) \times \ln(r)$$

This is in the form of the equation for a straight line:

$$y = mx + b$$

Where the identification of:

$$y \equiv \ln(\sin(\theta)), b \equiv \ln(\text{constant}), m \equiv -\text{power}$$

In an ideal situation, the power would be 2 (my results gave about 1.8).

In the spreadsheet, I fit this equation and then provide the plot for

$$\sin(\theta) = \text{Constant} \left[\frac{1}{r^{\text{Power}}} \right]$$

from both the data and the theoretical fit.

Error calculations and how I made the x and y error bars

On the spreadsheet I have done additional calculations. I estimated the errors for both separation and the resulting error in the sin of the angle. I repeated the fit with the maximum uncertainty and the minimum uncertainty in the separation to provide the uncertainty in the power and the constant, and also the % uncertainty in the error. I recommend not changing the estimated errors since the error bars (both x and y) are then not going to be set right. The way I made the x error bars is this: I obtained the java program for a ruler (from <http://noutash/freebies/nruler>) that I placed on the excel spreadsheet after calibrating the ruler scale to the length of the x-axis. I then wanted my x error bars to be of \pm the length of the estimated error in separation. In the icon choice for the data symbol, I picked the symbol that looked like a bowtie and made the width about .1 cm and changed the length (measured directly by the ruler) to be the calculated size based upon my ruler calibration. I then fixed the x axis scales so that this length would be preserved. Including the y-error bars was much easier since it can be done automatically. I am providing this as an example for your own future work. Note that if you resize the graph, the x-calibration will be incorrect so do not resize the graph.

Procedure:

At the beginning I would like to emphasize that your data may not be as good as mine. You might want to repeat the experiment several times (but quickly). I repeated mine about 5 times to get the data represented. One partner should make the measurements while the other writes them down. You will have 10 measurements total (@ 35,36,...,44). You will gently twitch the string at the ruler after moving the ping pong ball each time since the string tends to stick.

Record (and verify) the following:

PoPPBall: Position of the front of the base of the ping pong ball stand: it should be about **45** cm. This is adjusted so that the string, if lowered, would pass through the center of the ping pong ball. This is read from the meter stick on the table. Record your value: Do this after your measurements below.

PoPPBall= _____ cm

PoSBall: Position of the small ball at equilibrium. This is read from the metal ruler. If things are set correctly it would read **23**, but record your value. Do this after your measurements below.

PoSBall= _____ cm

Height from the ceiling to the top of the metal ruler: this is set to be 150 cm. However if you do not believe my measurement, you may measure and record this distance (trust me). I placed a black mark on the string at the top to the ruler to keep this correct. **Hint: trust me.**

H= _____ **150** cm

Distance between the top of the metal ruler to the center of the small ball: I have set this to be **23** cm but you can measure this to verify. On the spreadsheet I name this "ExtraHeight". Do this after your measurements below.

ExtraHeight= _____ cm

This is the way to make your measurements the quickest way: charge the system and move the ping pong ball holder to about 10 cm to the 35 cm mark (PoPPBall) so that L2 reads 35. I have arranged the system so that the front of the base reads 45 cm when the string would hang through the center of the ping pong ball and you should verify this and make adjustments as necessary. **Quickly and without breathing on your system** record the two numbers L1 and L2, and then move the Ping Pong base towards PoPPBall by 1 cm at a time. You should gently tip the thread at the ruler top to avoid snags. Do not move further than PoPPBall. This will give you 10 measurements. **Do this quickly and do not breathe on your system** since charge will change as a result of humidity.

The analysis

Your writeup for this lab will consist of your spreadsheet, together with your data submitted as a spreadsheet (in readable pdf format), and additionally your work should include a discussion of if your data shows an inverse square relationship and why and how well your experiment verifies Coulomb's law. It should also include a discussion of the procedure that you used to determine this. Also be sure to include in your discussion why it is important that the charge on the two balls stay constant while you are doing your experiment (this is, in fact, the major source of error here). Your power should automatically be calculated for you giving the experimental error (note that OpenOffice does this but gnumeric actually fails here for some reason). This is done with the OpenOffice "slope" function taken from a log-log fit. The percentage error will be reported relative to the expected power of 2. Although you will also obtain the constant, this is not useful for our purposes today, except to the extent that if it is not really a constant for your experiment, your results will be really really bad. Other than that, your lab writeup should be presented in the standard format for lab reports that you have already learned.