	Physics 250: Worksheet 3A	Name: ________________

Supplementary problems for electrostatics

1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m. Find the electric field at the origin.

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.

4. A sphere of radius a has a charge density which varies as $\rho = Q\frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

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1. Three identical charges each with q=1μC are arranged at the vertices of an equilateral triangle with a side of length 1m . Find the electric field at the origin.

Align your triangle so that one charge is located at (-L/2,0), the second is at (+L/2,0) and the third is at (0,a). At the origin, the electric field in the x-direction cancels. So you only need to calculate the field along the y axis from the third charge. We need to calculate a. This is given by:

$$a = \sqrt{L^2 - \left(\tfrac{L}{2}\right)^2} = L\sqrt{1-\tfrac{1}{4}} = L\sqrt{\tfrac{3}{4}} = \tfrac{\sqrt{3}L}{2}$$

This is:

$$\vec{r}_i = 0\hat{x} + \tfrac{\sqrt{3}L}{2}\hat{y} : \vec{r}_p = 0\hat{x} + 0\hat{y} : \vec{r}_{ip} = \vec{r}_p - \vec{r}_i = 0\hat{x} - \tfrac{\sqrt{3}L}{2}\hat{y}$$

$$\vec{E} = kq\frac{-\frac{\sqrt{3}L}{2}\hat{y}}{\left[\left(\frac{\sqrt{3}L}{2}\right)^2\right]^{3/2}} = kq\frac{-\frac{\sqrt{3}L}{2}\hat{y}}{\left[\left(\frac{3L^2}{4}\right)\right]^{3/2}} = -kq\frac{L}{L^3}\frac{\frac{\sqrt{3}}{2}}{\left[\frac{3}{4}\right]^{3/2}}\hat{y} = -\frac{kq}{L^2}\frac{0.866}{0.650}\hat{y} = -1.33\frac{kq}{L^2}\hat{y}$$

In our case at hand, we then have:

$$\vec{E} = -1.33\frac{kq}{L^2}\hat{y} = -1.196x10^4\,\hat{y}\,\tfrac{N}{C}$$

2. A square has charges at the following locations and values:
(1:1μ,-1,-1),(2:2μ,1,-1),(3:3μ,1,1),(4:4μ,-1,1).
Find the value of the vector electric field at the origin.
Form each of the vectors:

$$\vec{r}_{1p} = \hat{x} + \hat{y} : \vec{r}_{2p} = -\hat{x} + \hat{y} : \vec{r}_{3p} = -\hat{x} - \hat{y} : \hat{r}_{4p} = \hat{x} - \hat{y}$$

Form the magnitudes:

$$\left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = \left|\vec{r}_{4p}\right| = \sqrt{2}$$

Form the unit vectors:

$$\hat{r}_{1p} = \tfrac{\hat{x}+\hat{y}}{\sqrt{2}} : \hat{r}_{2p} = \tfrac{-\hat{x}+\hat{y}}{\sqrt{2}} : \hat{r}_{3p} = \tfrac{-\hat{x}-\hat{y}}{\sqrt{2}} : \hat{r}_{4p} = \tfrac{\hat{x}-\hat{y}}{\sqrt{2}}$$

Form E:

$$\vec{E}_p = \tfrac{k\mu}{2\sqrt{2}}\left\{(1-2-3+4)\hat{x} + (1+2-3-4)\hat{y}\right\} = \tfrac{k\mu}{2\sqrt{2}}\left\{0\hat{x} - 4\hat{y}\right\} = 12713\tfrac{N}{C}$$

3. Three charges are arranged as follows:
(1:1μ,-1,0),(2:-1μ,1,0),(3:1μ,0,1).
Find the vector electric field at the origin.
In this case, only charges 1 and 2 contribute to fields along the x direction and only charge 3 contributes to the field along the y direction. The electric field vectors are easily written:

$$\vec{r}_{1p} = \hat{x} : \vec{r}_{2p} = -\hat{x} : \vec{r}_{3p} = -\vec{y} : \left|\vec{r}_{1p}\right| = \left|\vec{r}_{2p}\right| = \left|\vec{r}_{3p}\right| = 1 : \hat{r}_{1p} = \hat{x} : \hat{r}_{2p} = \hat{y} : \hat{r}_{3p} = \hat{y}$$

Then the electric field is given by:

$$\vec{E}_p = k\mu\left\{-\hat{x} - \hat{x} - \hat{y}\right\} = k\mu\left[-2\hat{x} - \hat{y}\right] = \left\{-71980\hat{x} - 8990\hat{y}\right\}\tfrac{N}{C}$$

4. A sphere of radius a has a charge density which varies as $\rho = \rho_0 \frac{r}{a}$ inside and zero outside the sphere. Find the electric field inside and outside the sphere.

$$Q = \iiint \rho dV = \frac{4\pi}{a}\int_{r=0}^{r=a} \rho_0 r^3 dr = \frac{4\pi\rho_0}{a}\frac{a^4}{4} = \rho_0 \pi a^3 \Rightarrow \rho_0 = \frac{Q}{\pi a^3} \text{ so } \rho = \frac{Q}{\pi a^4} r$$

The electric flux is given by: $\Phi = \oint \vec{E} \bullet d\vec{A} = E\oint d\vec{A} = E4\pi r^2$

Inside the sphere, the enclosed charge is given by:

$$Q_{enc} = 4\pi\int_{r=0}^{r} \rho r^2 dr = 4\pi\left[\frac{Q}{\pi a^4}\right]\int_{r=0}^{r=r} r^3 dr = 4\pi\left[\frac{Q}{\pi a^4}\right]\left[\frac{r^4}{4}\right] = Q\left(\frac{r}{a}\right)^4$$

On the Gaussian surface, we then have:

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}\left(\frac{r}{a}\right)^4 \Rightarrow \vec{E}_{in} = \frac{Q}{4\pi\varepsilon_0}\frac{r^2}{a^4}\hat{r}$$

Outside the sphere, the enclosed charge is Q. We then have:

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E}_{out} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Note that at the surface of the sphere, the two solutions are equal.

5. A parallel plate capacitor has a total charge Q=1μC on one plate and –Q=-1μC on the other plate. The plates have a cross sectional area of 1 m^2 and are separated by a distance of 0.1 m. What is the value of the electric field near the center of the capacitor?

The surface charge density is given by $\sigma = \frac{Q}{A} = 1\frac{\mu C}{m^2}$

Outside the capacitor note that the electric field is zero.
Choosing a cylindrical Gaussian surface of area A’ we have that between the plates, the electric flux through the ends of the cylinder is given by:

$$EA' = \frac{\sigma A'}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{1x10^{-6}}{\left(8.85x10^{-12}\right)} = 1.13x10^5\,\tfrac{N}{C}$$